POLYTECHNIC OF NAMIBIA

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CIVIL ENGINEERING DEPARTMENT

WATER ENGINEERING (WAE 3300)

Fluid properties

Mass related definitions

Density

Mass density is the mass of an object, substance, fluid, in ratio to its volume.

Because the volume changes with temperature (T), it is important to always indicate the reference temperature for very accurate work.

(Note: During class test or assignments, tutorials, you are allowed to just pick the "list-density" unless otherwise specified.)

The density is symbolized by "rho" = ρ and is the ratio of mass to volume. Its dimensions are: **[kg/m^3]**

In formula: **ρ = m/V**

Some useful values to refer back to:

Table of different fluid densities at standard temperature:

ρ_{water}	1000	Kg/m^3
$\rho_{mercury}$	13600	Kg/m^3
ρ_{oil}	800-930	Kg/m^3
$\rho_{seawater}$	1027	Kg/m^3
$\rho_{steam(100°C)atm.pr.}$	0,598	Kg/m^3
$\rho_{steam(200°C)atm.pr.}$	0,467	Kg/m^3
$\rho_{steam(300°C)atm.pr.}$	0,384	Kg/m^3
$\rho_{mild\ steel}$	7850	Kg/m^3
$\rho_{alcohol}$	700-750	Kg/m^3

Table for water density at different temperatures

$\rho_{water(0°C)}$	999,8	Kg/m^3
$\rho_{water(4°C)}$	1000	Kg/m^3
$\rho_{water(10°C)}$	999,6	Kg/m^3
$\rho_{water(20°C)}$	998,2	Kg/m^3
$\rho_{water(30°C)}$	995,6	Kg/m^3
$\rho_{water(40°C)}$	992,2	Kg/m^3
$\rho_{water(50°C)}$	988	Kg/m^3
$\rho_{water(60°C)}$	983,2	Kg/m^3
$\rho_{water(80°C)}$	971,8	Kg/m^3
$\rho_{water(100°C)}$	958,3	Kg/m^3

Specific Volume

The specific volume is just the inverse of density, it is the ratio of volume to mass, it symbolises the volume of 1 kg of fluid or substance.

The dimension of the specific volume is m^3/kg.

In formula: **$v=V/m = 1/\rho$** in **$[m^3/kg]$**

Relative Density

The relative density is the ratio of the density of any fluid or object to the density of water at 4°C.

This is a dimensionless number.

In formula: **$S=\rho_{fluid, object}/\rho_{water(4°C)}$** **[ ]**

For example: mercury's relative density S_{Hg}= 13600/1000 = 13.6

Specific Weight

The specific weight of an object, fluid, substance is the ratio of the weight [W] of an object, fluid, substance to the volume [V] of 1 m^3.

The dimension of the specific weight is **N/m^3**. It can also be expressed as mass times gravitational constant [g] divided by volume.

In formula: **$\gamma = W/V = mg/V = \rho g$ [N/m^3]**

Note: Remember for future computations the formula **$\gamma = \rho g$ [N/m^3]**

Viscosity

A fluid at rest cannot resist shearing forces but once it is in motion shearing forces are set up between layers of fluid moving at different velocities. The viscosity of the fluid determines its ability to resist these shearing stresses

The Coefficient of Dynamic Viscosity □ (Gk., mu) is defined as the shear force per unit area required to drag one layer of fluid with unit velocity past another layer unit distance away from it in the fluid. SI

unit, $N\text{-}s/m^2$ or kg/m-s.

Kinematic Viscosity ν (Gk., nu) is the ratio of dynamic viscosity to mass density

The SI unit is m^2/s.

Variation of Viscosity with Temperature.

The viscosity,μ of liquids decreases with increase of temperature

Summary

- Viscosity is a measure of the ease with which molecules move past one another
- It depends on the attractive force between the moleculees
- It depends on whether there are structural features which may cause neighboring molecules to become "entangled"
- Viscosity ***decreases*** with increasing temperature - the increasing kinetic energy overcomes the attractive forces and molecules can more easily move past each other

Surface Tension

Within the body of a liquid a molecule is attracted equally in all directions by the other molecules surrounding it, but at the surface between liquid and air the upward and downward attractions are unbalanced. The liquid surface behaves as if it were an elastic membrane under tension. This surface tension is the same at every point on the surface and acts in the plane of the surface normal to any line in the surface. Surface tension is not affected by the curvature of the surface, and it is constant at a given temperature for the surface of separation of two particular substances. Increase of temperature causes a decrease of surface tension.

Surface tension causes drops of liquid to tend to take a spherical shape and is also responsible for capillary action which causes a liquid to rise in a fine tube when its lower end is inverted in a liquid which wets the tube. If the liquid does not wet the tube it will be depressed in the fine tube below the surface outside.

If θ is the angle of contact between liquid and solid, upward pull due to surface tension = $\sigma\pi d \cos\theta$ where d = diameter of tube.

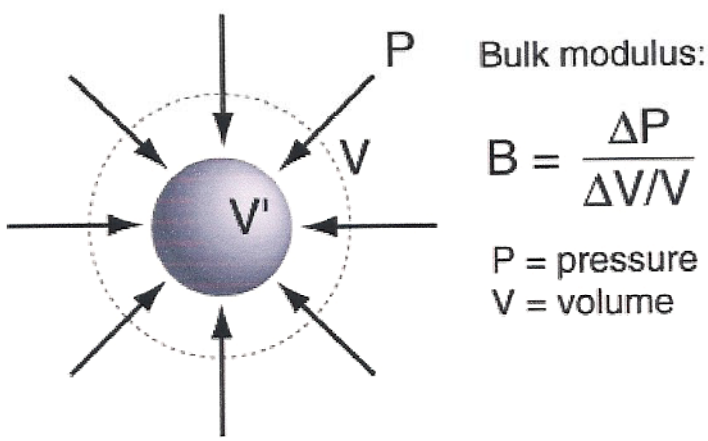
Putting h = height liquid is raised and w = sp. wt of liquid

weight of liquid raised = $w*\pi/4*d^2*h$

so that = $\sigma\pi d \cos\theta = w*\pi/4*d^2*h$

Bulk Modulus

The bulk elastic properties of a material determine how much it will compress under a given amount of external pressure. The ratio of the change in pressure to the fractional volume compression is called the bulk modulus of the material.



A representative value for the bulk modulus for water is

$$B_{water} = 2.2*10^9 \frac{N}{m^2}$$

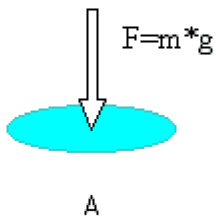
The reciprocal of the bulk modulus is called the compressibility of the substance. The amount of compression of solids and liquids is seen to be very small.

Compressibility is the fractional change in volume per unit increase in pressure. For each atmosphere increase in pressure, the volume of water would decrease 46.4 parts per million. The compressibility k is the reciprocal of the Bulk modulus.

Pressure in fluids

Pressure & Pressure heads

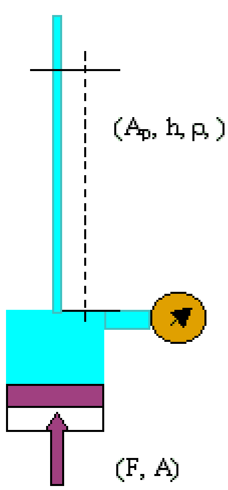
Pressure [**p**] or **pressure density** is defined as the force [**F**] per unit area [**A**] perpendicular to the force.



$$p = F/A \quad [Nm^{-2}]$$

The dimension is either **Newton per square meter** or **Pascal**

Consider the following piston-cylinder arrangement which is filled with a liquid. If we apply a force [F] on the plunger, the liquid level in the thin pipe with cross sectional area [A_P] will rise to a certain height [h]. The pressure will be shown on the gauge [p]



$$p = F/A = (\rho * g * A_P * h / A_P) = \rho * g * h$$

Where $\rho * g * A_P * h$ is the weight of the liquid in the pipe.

A weight is always interpreted as a force.

To convert pressure head [h] from fluid A to fluid B ➔ $h_A=h_B*\rho_B/\rho_A$

Remember: Because the pressure head depends on the density of the fluid, it is always necessary to specify the fluid used and the temperature measured.

Table of different fluid densities at standard temperature:

ρ_{water}	1000	Kg/m^3
$\rho_{mercury}$	13600	Kg/m^3
ρ_{oil}	800-930	Kg/m^3
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$\rho_{water(100°C)}$	958,3	Kg/m^3

Absolute Pressure

The pressure and pressure head discussed earlier is known as **gauge pressure**.

It is the amount by which the pressure is higher or lower than atmospheric pressure.

The sum of atm. pressure and gauge pressure is defined as **absolute pressure**.

In formula: $\mathbf{p_{abs}=p_{gauge}+p_{atm}}$

Sign handling: **p_{abs} is always positive due to positive atmospheric pressure**, which can vary from day to day and from place to place (height above sea level). **p_{gauge} can be positive or negative**.

With **$p_{abs}=0$** we find an **absolute vacuum**, where no molecules can cause pressure, so there will be no collision of molecules.

A partial vacuum can be specified in three different way:

- …a vacuum of 65 kPa
- …pressure of –65 kPa
- …a pressure of 35 kPa **absolute**, if atm. pressure is round about 100 kPa

(Note: Alternatively a partial vacuum can be expressed in [L] fluid column (Temp).)

Atmospheric Pressure

… Is the pressure caused by the atmospheric influence.

It is the pressure > rho_{air} * g * $height_{above\ surface\ level}$ <

where the "height" is the total height of an air column from surface level up to the very outer atmosphere.

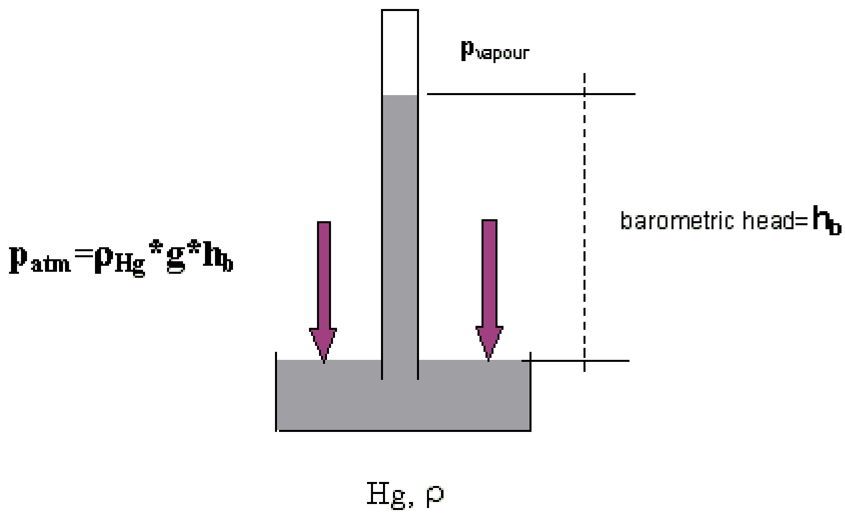
As this is pretty difficult to measure with all three parameters changing as we leave the earth's surface,

- the density ρ of air is not constant (the higher we climb up from sea level, the less dense the air becomes),
- the gravitational constant $\mathbf{g}$ also changes (see: weightlessness) ,
- and we would not know where to precisely stop to measure the height $\mathbf{h}$ either.

… there must a different way to measure atmospheric pressure right on the surface under consideration.

Thus atmospheric pressure at earth level is most commonly measured with a **Mercury Barometer**.

Look at the following setup, outlining a **Mercury Barometers**, used because of the **high ρ of mercury**.



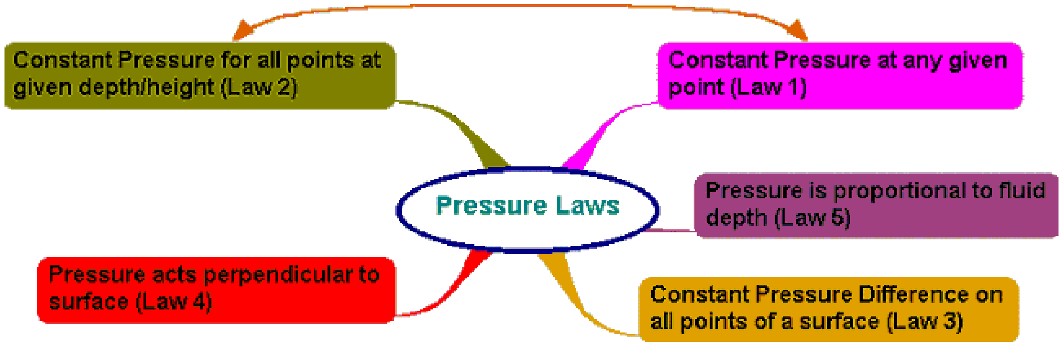
To make a simple device like this, you completely fill a one end sealed glass tube with mercury (Note: Mercury is poisonous) and turn it upside down in a bowl filled with mercury. Do not allow air to enter.

The mercury level will drop until the atmospheric pressure is just enough to sustain the column.

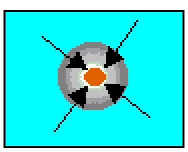
The standard atmospheric pressure head of mercury is 760mm. With **p = ρgh** : **13600 kg/m³** x 9.81 x 0.760m = 101396 Pa or **101,396 kPa**

Hydrostatic Pressure laws

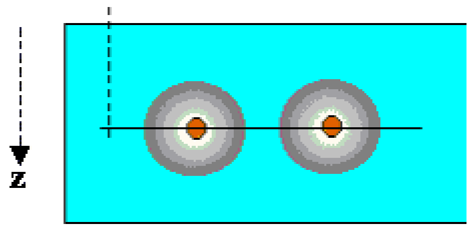
Hydrostatic Pressure Laws



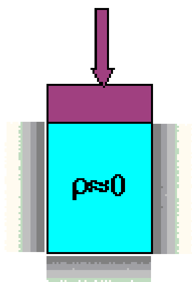
- **The pressure at a certain point (x,y,z) in a fluid which is at rest, is the same in all directions.**



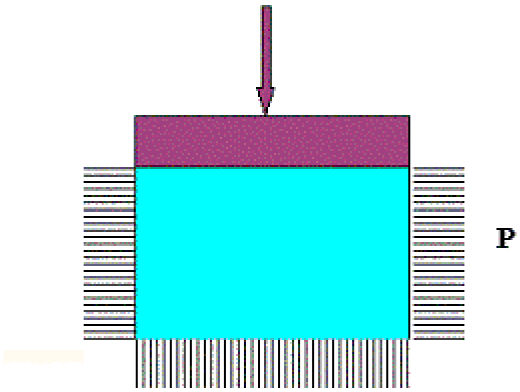
- **$_1$,y_1,z_1 and x_2, y_2,z_2) in a fluid at rest are at the same height (z_1=z_2), and one can move from point (1) to point (2) without leaving the fluid, then the two points receive the same pressure.**

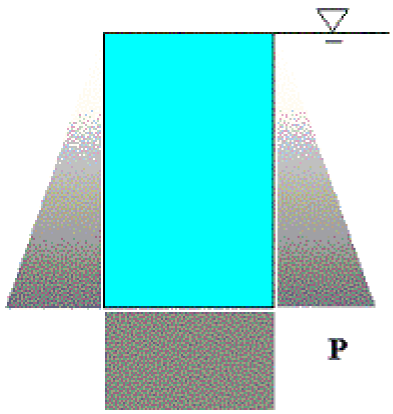


- If a pressure is applied on the surface of a fluid, the same amount of pressure is transmitted in all directions in the fluid (Pascal's Law).



- Pressure is always perpendicular to the surface it acts upon.





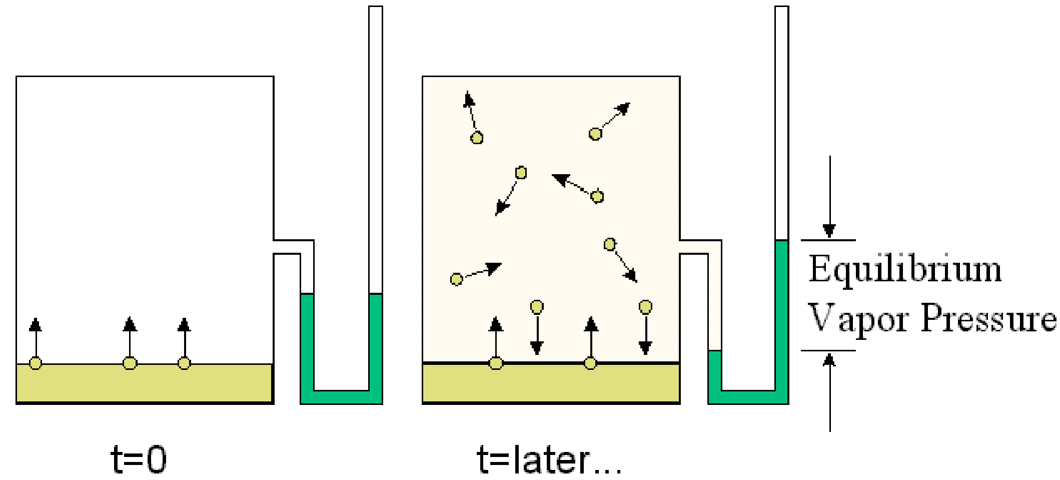
Vapour Pressure

Imagine a sealed container partly filled with a fluid **(ρ, T, p)**. Because the molecules of the fluid are in continuous motion, some on the surface escape into the space above and rejoin later again.

The liquid and vapour are therefore continuously in equilibrium for a given temperature (T). The molecules in the space above the liquid are exerting pressure on the container. This pressure is called **vapour pressure**.

The higher the temperature in the container the higher the vapour pressure.

If on the other hand the pressure on a liquid is lower than the vapour pressure for a specific temperature, the liquid will boil.



(Find every day examples for qualitative discussion, ... pressure cooker, boiling water at an altitude of 10.000m above sea level, ...)

Cavitation

In certain machines (centrifugal pumps) it could happen that the water pressure in certain regions in the pump drop lower than vapour pressure and cause the water to boil forming water vapour bubbles. When the mixture – water and vapour bubbles – move to a region of higher pressure, the bubbles collapse and suddenly condense back to water again. The small shock waves caused by collapsing bubbles cause large local stress on the surface of parts of the pump (many MPa). These stresses may lead to material failure on the surfaces in turn causing erosion of the pump parts. The erosion affects the pump detrimentally. This phenomenon is called **cavitation**, and must be avoided by ensuring that the pressure never falls too low in the pump.

Measurement of pressure

Why do we need to measure pressure?

Why do we need to quantify pressure?

Pressure and especially sudden pressure changes always cause stress on materials. For knowing about stress on materials and pressure, or pressure density to act, a surface is needed.

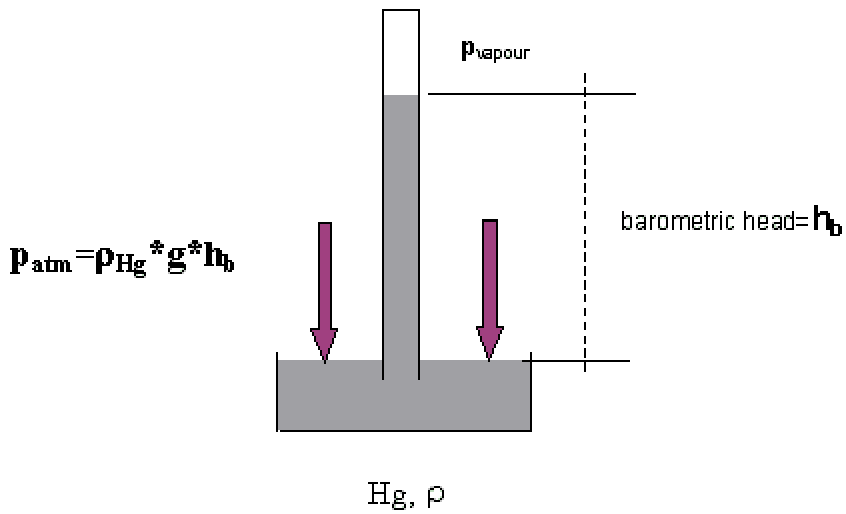
But, pressure may also acts as a resistance.

In addition, we all know about pressure in water pipe lines. Here it is interesting to know about the pressure drop along a pipe line.

...

Measurement of Pressure

Barometers are used to measure atmospheric pressure and the most common type is the mercury barometer due to the high ρ of mercury.



To make a simple device like this, you completely fill a one end sealed glass tube with mercury (Note: Mercury is poisonous) and turn it upside down in a bowl filled with mercury. Do not allow air to enter.

The mercury level will drop until the atmospheric pressure is just enough to sustain the column.

The standard atmospheric pressure head of mercury is 760mm. With **$p = \rho gh$** : 13600 kg/m³ x 9.81 x 0.760m = 101396 Pa or **101,396 kPa**

There are other systems in use to measure atmospheric pressure such as the **aneroid barometer**, built up like a small tin with a flexible corrugated plate, sealed all around, partially evacuated. Airplanes use such devices to measure height differences. A more or less complicated lever and gear system causes a needle to be deflected. Such systems operate with a vacuum close to zero, suitable even with temperature changes.

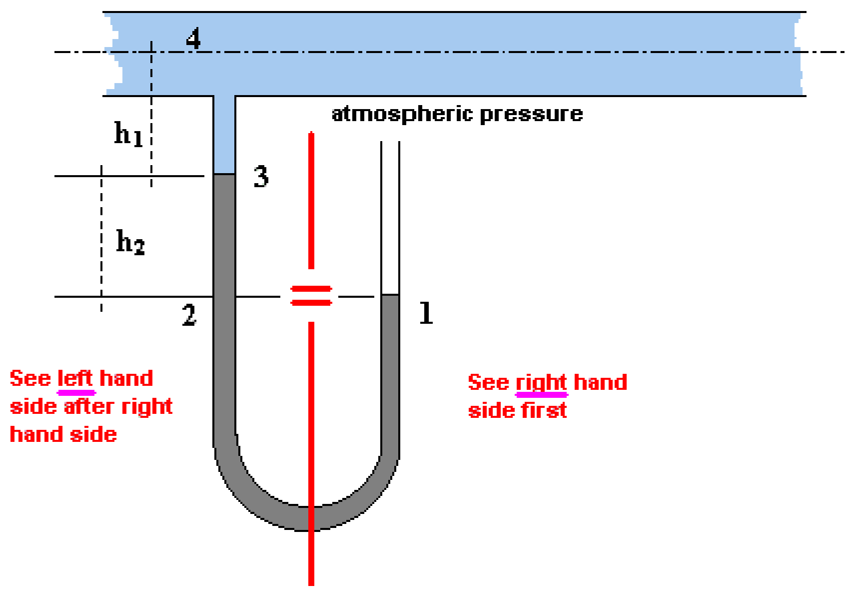
The **Bourdon pressure gauge** works with a circular "question-mark" like bent, flattened, hollow tube to which a gear and needle setup is connected. Pressure differences cause the structure to uncurl slightly. Zero reading of course is obtained, when the pressure inside and outside are equal.

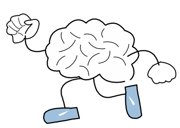
The Piezometer or StandPipe is simply a vertical transparent tube, connected to the container of which the pressure is to be measured. Recall the relationship between **ρ, g, h ➔ p**. The disadvantage for such a setup is that high pressures need very long tubes which could be impractical.

U-Tube manometer

The U-Tube Manometer can be used to measure pressures higher or lower than atmospheric pressure, $\mathbf{p_{atm}}$. A U-tube manometer contains manometer fluid with a density ρ_m which is greater than the density of the fluid in the pipe ρ_f.

Look at the following arrangement outlining a U-tube manometer. (Note: the disconnected side of the U-tube is open to atmospheric pressure.)





Always write down first what you know as a logical and true statement!

$\mathbf{p_{(1)} = p_{atm} = p_{(2)}}$ **(!)**

:➔ $\mathbf{p_{(2)} = p_{(4)abs} + \rho_f g h_1 + \rho_m g h_2}$

(Sum of all pressure components in point$_{(2)}$ is balanced with $\mathbf{p_{atm}}$)

- **substitute p_{atm} for $p_{(2)}$ into the above equation**
- **re-arrange the equation to solve for $p_{(4)abs}$**

:➔ $\mathbf{p_{(4)abs} = p_{atm} - \rho_f g h_1 - \rho_m g h_2}$

----

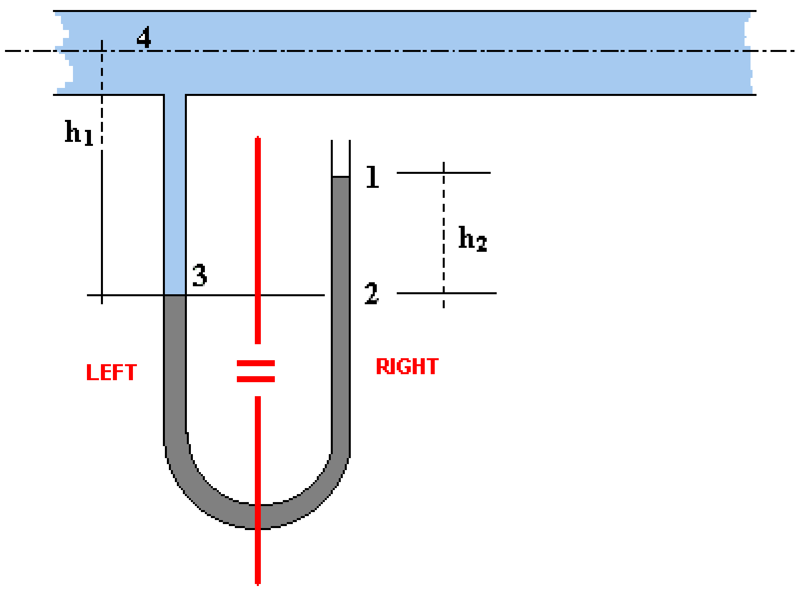
interested in the gauge pressure at point$_{(4)}$:

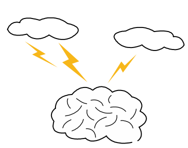
- **simply subtract the atmospheric pressure p_{atm}** (remember: $p_{abs}=p_{gauge}+p_{atm}$)

:➔ $p_{(4)gauge}=-\rho_f g h_1 - \rho_m g h_2$

(If the fluid is **air** , then the term $\rho_f g h_1$ becomes so small that it can be neglected.)

Let us look at the same U-Tube Manometer, with a different reading. This time the manometer fluid climbs up to the open top on the right.





Always write down first what you know as a logical and true statement!

$p_{(1)}=p_{atm}$ (!)

:➔ $p_{(2)}=p_{(3)}= + \rho_m g h_2 + p_{atm}$ (Note: Sum of all pressure components in point$_{(2)}$ includes $\mathbf{p_{atm}}$)

- **now look at the left hand side**

:➔ $p_{(3)} = p_{(4)abs} + \rho_f g h_1$

- **substitute $\rho_m g h_2 + p_{atm}$ for $p_{(3)}$**

:➔ $\rho_m g h_2 + p_{atm} = p_{(4)abs} + \rho_f g h_1$

- **re-arrange the equation to solve for $p_{(4)abs}$**

:➔ $p_{(4)abs} = +\rho_m g h_2 + p_{atm} - \rho_f g h_1$

----

interested in the gauge pressure at point$_{(4)}$:

- **simply subtract the atmospheric pressure p_{atm}** (remember: $p_{abs} = p_{gauge} + p_{atm}$)

:➔ $p_{(4)gauge} = + \rho_m g h_2 - \rho_f g h_1$

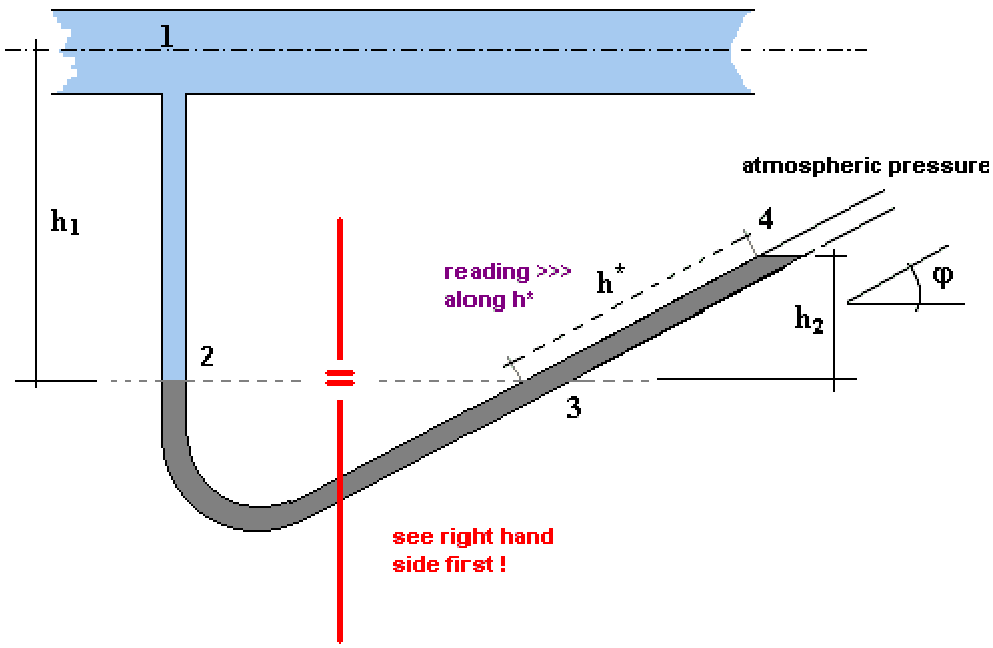
(If the fluid is **air** , then the term $\rho_f g h_1$ becomes so small that it can be neglected.)

Inclined

To improve the sensitivity of a manometer, one arm can be bend downwards at an angle φ. The fluid level movement in the **inclined arm of a manometer**, here inclined U-Tube-Manometer, is much greater than in a vertical arm, therefore small pressure changes can be readily detected.

The pressure difference between point$_{(1)}$ and point$_{(4)}$ follows again $\rho_b g h_2$, which equals $\rho_b g h^* \sin\varphi$, with $h_2 = h^* \sin\varphi$.

Look at the following **inclined U-Tube-Manometer** setup and derive for the absolute pressure and the gauge pressure at point$_{(1)}$.



Always write down first what you know as a logical and true statement!

$_{(4)}=p_{atm}$ **(!)**

:➔ $p_{(2)}=p_{(3)}= + \rho_m g h^* \sin\varphi + p_{atm}$ (Note: Sum of all pressure components in point$_{(3)}$ includes p_{atm})

- **now look at the left hand side**

:➔ $p_{(2)}=p_{(1)abs} + \rho_f g h_1$

- **substitute $\rho_m g h^* \sin\varphi + p_{atm}$ for $p_{(2)}$**

:➔ $\rho_m g h^* \sin\varphi + p_{atm} = p_{(1)abs} + \rho_f g h_1$

- **re-arrange the equation to solve for $p_{(1)abs}$**

:➔ $p_{(1)abs}= +\rho_m g h^* \sin\varphi + p_{atm} - \rho_f g h_1$

----

interested in the gauge pressure at point$_{(1)}$:

- **simply subtract the atmospheric pressure p_{atm}** (remember: $p_{abs}=p_{gauge}+p_{atm}$)

:➔ $p_{(1)gauge}= + \rho_m g h^* \sin\varphi - \rho_f g h_1$

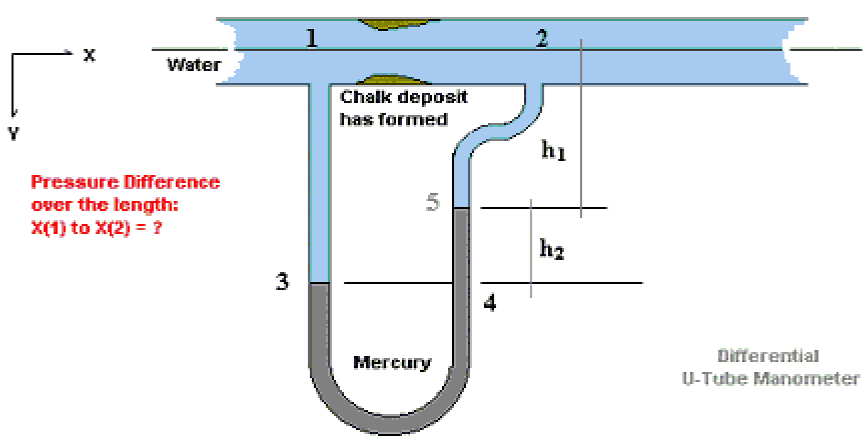
(If the fluid is **air** , then the term $\rho_f g h_1$ becomes so small that it can be neglected.)

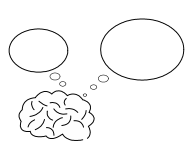
Differential U-Tube

The **Differential U-Tube Manometer** is used where pressure difference between two points must be measured. The pressure can be much higher or lower than atmospheric pressure. The manometer must have a fluid density greater than the fluid in the pipe.

Have a look at the following setup outlining a differential U-Tube Manometer. To show a reasonable pressure difference a chalk deposit simulates the venturi effect.

The deposits represent a resistance to the flow, thus pressure has to be reduced.





write down first what you know as a logical and true statement!

$p_{(3)}=p_{(4)}$ (!)

- **now look at the right hand side first**

:➔ $p_{(4)} = p_{(2)} + \rho_f g h_1 + \rho_m g h_2$ (Note: Sum of all pressure components in point$_{(4)}$ this time excludes p_{atm})

- **now look at the left hand side**

:➔ $p_{(3)} = p_{(1)} + \rho_f g h_1 + \rho_f g h_2$

- **substitute $p_{(1)} + \rho_f g h_1 + \rho_f g h_2$ for $p_{(3)}$ in your first equation**
- **substitute $p_{(2)} + \rho_f g h_1 + \rho_m g h_2$ for $p_{(4)}$ in your first equation**

:➔ $p_{(1)} + \rho_f g h_1 + \rho_f g h_2 = p_{(2)} + \rho_f g h_1 + \rho_m g h_2$ **(you can delete the term $\rho_f g h_1$)**

:➔ $p_{(1)} + \rho_f g h_2 = p_{(2)} + \rho_m g h_2$

- **re-arrange the equation to solve for Δp (the pressure difference between $p_{(1)} - p_{(2)}$)**

:➔ $p_{(1)} - p_{(2)} = +\rho_m g h_2 - \rho_f g h_2$ **(➔RESULT)**

if ρ_f is close to zero

: ➔ $\Delta p = +\rho_m g h_2$ **(for example if the fluid$_f$ is air)**

---

interested in terms of pressure head:

- **simply divide (RESULT-formula) above by $\rho_f g$**

$(p_{(1)} - p_{(2)}) / \rho_f g = (+\rho_m g h_2 - \rho_f g h_2) / \rho_f g$

: ➔ $(p_{(1)} - p_{(2)}) / \rho_f g = ((\rho_m / \rho_f) - 1)\, h_2$

with $(\rho_m/\rho_f) = S_m$ = relative density of the manometer fluid

(Note: this makes sense especially when the fluid is water, then $S_m = 13.6$, but not if the fluid is air)

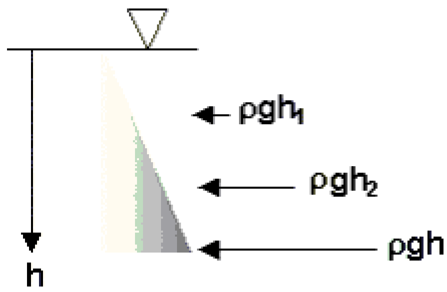
Hydrostatic forces

Vertical flat surface

See also: Vertical

The pressure of a fluid causes a thrust (➔Force) to be exerted on every part of any surface with which the fluid is in contact. The deeper a surface is submerged into the fluid, the more increases the pressure and the bigger becomes such force.

Recall the >Hydrostatic Pressure Laws< and F=p*A



Look at the above pressure distribution diagram. We recognize the distribution diagram as a triangle. >>>Why?

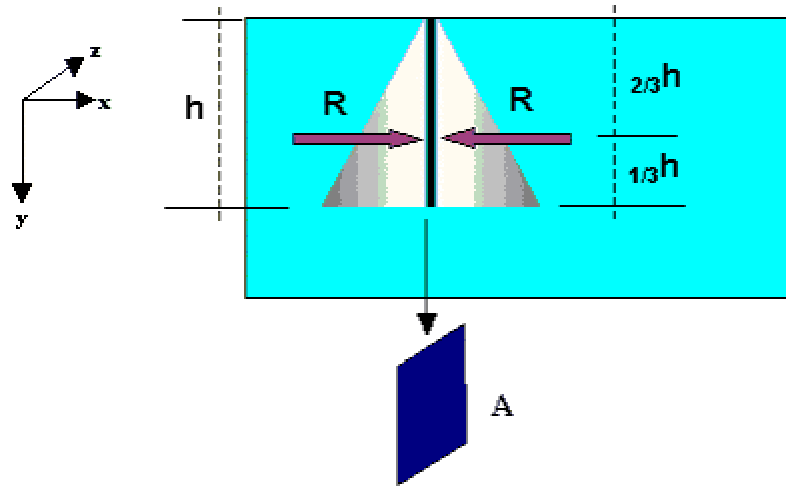
With ρ and g being constant, and an increase in height/depth [h] to the first power, the increase of ρgh must be linear, thus it must be a straight line, forming a triangle from h=0 to h.

The individual forces distributed over an area [A] have – in general – a resultant force [**R**].

The determination of this resultant's magnitude, direction, and position is frequently important.

Now we want to understand how and where such resultants act.

Imagine a rectangular flat surface (blue coloured), wholly, vertically submerged in a liquid in equilibrium and just "photograph" this diagram, its explanation will follow below.



ρ**gh** is the pressure for any given height/depth. And we recall a force **[F]** being ρ**ghA**

Primarily we are interested in two points on the surface of our plate:

- The Centroid (y,z), indicating the center of gravity (x,y,z) on the submerged plate/plane surface
- The Center of Pressure (y_R,z_R), the point of action of our resultant [R] (x_R).

Obviously the two points differ in position due to an increase in pressure with increasing depth **[h]** for the resultant **[R]**, and the stable, given form of our plate with a clearly defined center of gravity. Imagine the plate not being submerged wholly, then the above said will become clear.

The center of pressure [h_p] is always positioned lower in the fluid than the centroid [h_c], except when the surface is horizontal.

As **[➔h]** is positive when drawn in vertical downward direction, **$h_p > h_c$.**

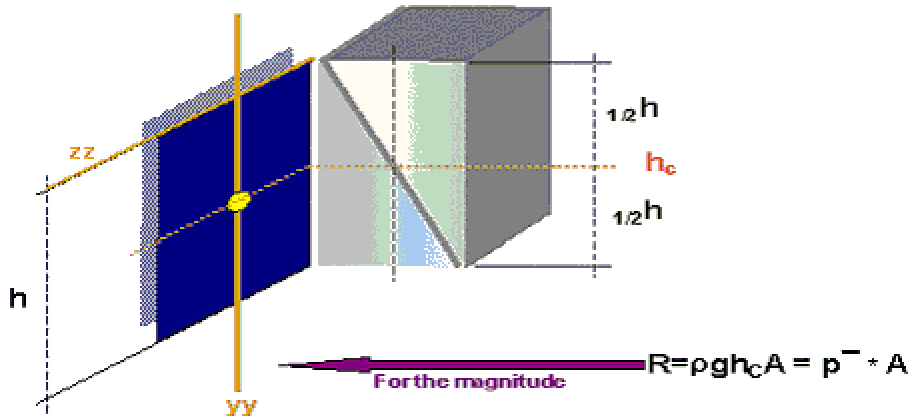
………………………………………………………………………………

Now let us call for the **magnitude** of **[R]** first:

Before doing so, we have to attend to the pressure, in fact to the average pressure on our **rectangular, flat, vertically submerged surface.**

Average pressure ($\bar{p}$) = (top pressure + bottom pressure)/2

…the half of the sum of top pressure plus bottom pressure. See the diagram below.



…or *in formula*: **the average pressure $\bar{p} = \rho g h_C$ at the centroid [h_C] of our surface under consideration.**

For mathematical details and full explanation, please refer back to your **Mathematics 1 and 2**, your **AppliedMechanics 1**,

and/or to B.S.Massey, Mechanics of fluids, ELBS, 1990, **and** C.F.Meyer, Water Engineering, Technikon of Pretoria, 1995

Having calculated the average pressure, we now follow the known equaition of:

F=pA ➔ to find the magnitude of the resultant force **$R=\rho g h_C A = \bar{p} * A$.**

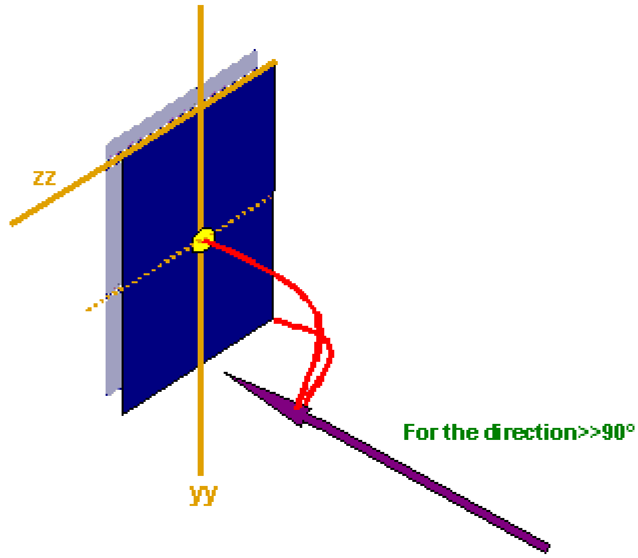
The **direction** of our resultant force is defined by Hydrostatic Pressure Law 4:

Recall the Hydrostatic Pressure Laws

The resultant force [R] points perpendicular to the surface point under consideration.

For inclined surfaces, the same is valid, but we will have a more detailed look later.

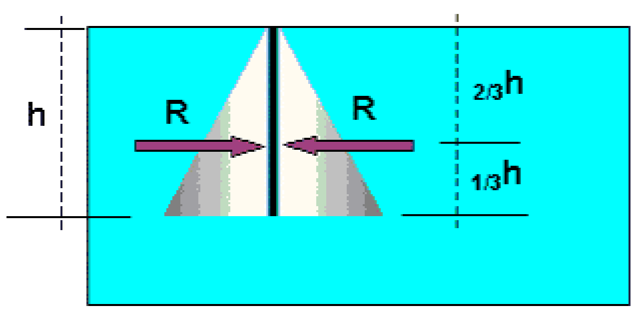
See the following diagram:



The **position/line of action** **of the resultant force on our rectangular vertical surface is defined as running through the centroid of the pressure distribution diagram (ρ=constant):**

Recall: finding the centroid of different surface shapes **Applied Mechanics (I,II)**

See the following diagram to understand the line of action.



The depth **[h*]** (measured down the plane) of the center of pressure equals ➔

h* = $\bar{h}$ + (Second moment of the area)/(First moment of area)

in Formula: **$h^* = \bar{h} + I/A\bar{h}$**

or $h^* = \bar{h} + k^2/\bar{h}$ …in terms of k^2

or $h^* = \bar{h} + k^2 \sin^2\varphi / \bar{h}$ …in terms of k^2 and for $\varphi = 90°$ ($\sin(90°) = 1$), vertically submerged

The formula with $h^* = \bar{h} + k^2 \sin^2\varphi / \bar{h}$ will be dealt with later, when we look at inclined surfaces, where three sets of parameters are known: the height/depth and the dimensions of the surface and the angle at which the surface is positioned.

Here $\bar{h}$ is the average height/depth of the submerged surface under consideration, or the **(h)position of the centroid** of the **submerged surface**.

The second moment of area is also known as the Moment of Inertia of an area and is always positive resulting in $h^* > \bar{h}$ for $\varphi = 90°$, remember, here we are talking of **vertically** submerged surfaces.

$h^* = \bar{h}$ is only valid for horizontally submerged surfaces ($\sin(0°) = 0$), then the term $k^2 \sin^2\varphi / \bar{h}$ in the above equation becomes zero.

(Note: XX-axis is normally defined as the axis along a beam or wall, thus yy-axis and zz-axis describe the cross-sectional area of a beam or structure under consideration.)

rectangular area with the yy-axis through the centroid parallel to the base line $I_{yy} = bd^3/12$

For a circular area with the yy-axis through the centroid $I_{yy} = \pi d^4/64$, or $I_{yy} = \pi r^4/4$

Such formulas can also be written in the form $I_{yy} = Ak^2$ or $k^2 = I_{yy}/A$

with k being the radius of gyration, solved to $k = \sqrt{(I_{yy}/A)}$

See applied **Mechanics 1**

Theorem of parallel axis

$I_{\chi\chi} = I_{xx} + A\ell^2$

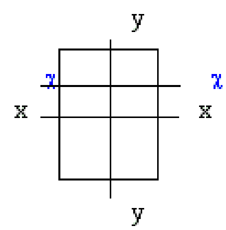
$= Ak^2 + A\ell^2$

$= A(k^2 + \ell^2)$

Mmt of Inertia / Second Mmt of Area

$I_{xx} = Ak_x^2$

$bd^3/12 = bd \,*\, d^2/12$



$\ell = |\chi - x|$, the distance between xx and χχ

Example to highlight the Theorem of parallel axis

We have a rectangular surface with b=10 cm and d=20 cm. Please compute the following parameters:

a) Area = ***?***

Centroid C_y, C_x and k_x = ***?***

Ixx and Iyy is asked too

The distance of **l** is 5cm.

b) compute **Iχχ**

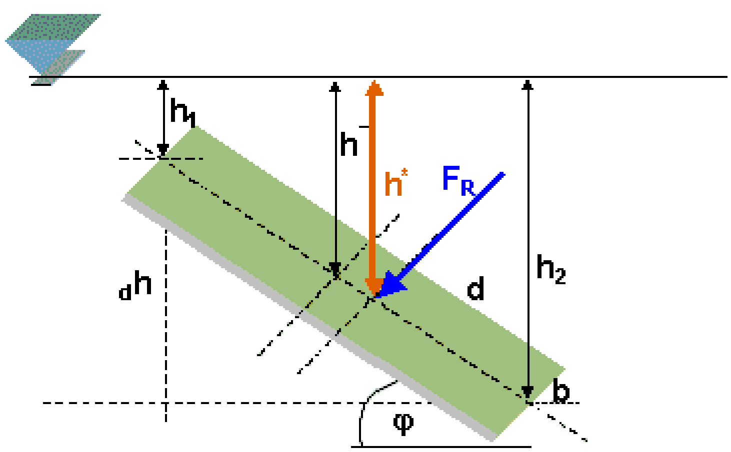
Solutions

- A = bd = 0.1m x 0.2m = 0.02 m²
- Cy = d/2 = 0.1 m
- Cx = b/2 = 0.05 m
- k_x = sqrt(d²/12) = 0.0577 m
- Ixx = bd³/12 = 0.0000667 m4
- Iyy = db³/12 = 0.0000167 m4
- $I_{\chi\chi}$ = Ixx + A(**l** ²) =0,000667 +0.02(0.05²) = 0.0001167m4

Inclined surface

See also: Inclined

When a flat surface is submerged into a fluid at rest, the hydrostatic force on it will be determined by the height/depth, the size and shape, and the angle at which it is positioned.



Look at the setup above and note the different heights, the angle and the shape of our surface with **b**=base and **d**=height of our rectangular flat plate.

We are primarily interested in three parameters of the resultant force F_R.

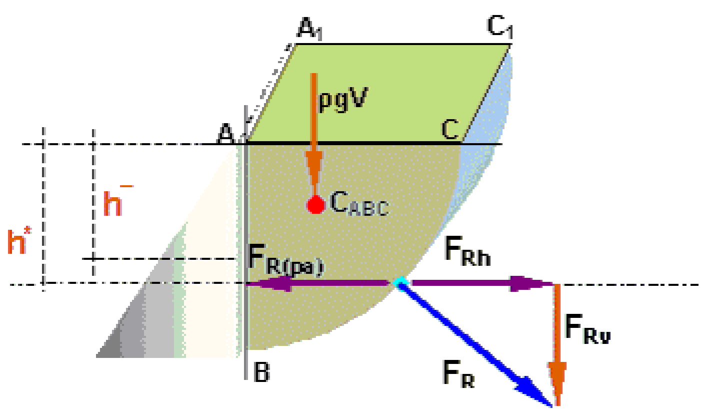
Direction ➔ perpendicular to the surface (90°-φ)

Magnitude ➔ $F_R = \rho g \bar{h} A$ **(the same formula as previously used for horizontal and vertical surfaces,** $\bar{h} = (h_1+h_2)/2$ or $({}_d h/2) + h_1$.)

Position ➔ $h^* = \bar{h} + (k^2 \sin^2\varphi)/\bar{h}$ with ${}_d h/d = \sin\varphi$, $k^2 = I/A$, $A = bd$, $I = bd^3/12$ (for rectangular surface)

Curved surface

Hydrostatic forces on curved surfaces



On curved surfaces the resultant hydrostatic force can be resolved into a horizontal and a vertical component **[F_{Rh}]** and **[F_{Rv}]**. The two components are then combined to the resultant force **[F_R]** and the direction φ.

Consider a container with a circularly curved side and four flat sides, the front profile of the container is a quarter circle with a 90° angle and two equal sides. The container is shown below and with *AC* being horizontal, and *AA_1*AB* being a vertical plane surface, the container is completely filled with a fluid **[ρ]**. The hydrostatic force **[F_R]** is resolved in two components.

On the vertical rectangular surface $AA_1 * AB = \mathbf{A}$ on the left hand side of our container, a hydrostatic force $[F_R]_{(pa)}$ acts with the magnitude $\rho g \bar{h} A$, at a position $h^* = \bar{h} + (k^2 \sin^2\varphi)/\bar{h}$ with $k^2 = d^2/12$ and $\varphi = 90°$, which is at 2/3(AB) due to a rectangular area **, also called the projected area (pa).** This force $[F_R]_{(pa)}$ on the left hand side of our container has to be balanced in opposite direction with a force of **equal magnitude and line of action** and is named $[F_{Rh}]$ on the curved surface.

$[F_{Rh}] = [F_R]_{(pa)}$

The vertical component $[F_{Rv}]$ is equal to the weight of the fluid and its line of action passes through the center of gravity of the fluid body.

The magnitude is $[F_{Rv}] = \rho g V$. **(Note: V=Volume of fluid displaced by the curved body.)**

The magnitude of the resultant force $[F_R]$ is found through adding the two vectors by $[F_R] = SQRT([F_{Rv}]^2 + [F_{Rh}]^2)$. (Pythagoras)

The direction is found with $\varphi = \tan^{-1}([F_{Rv}]/[F_{Rh}])$

The position is found on the intersection with the line of action of $[F_{Rh}] = -[F_R]_{(pa)}$ and the curved surface.

Note: If the curved surface's profile is circular, the line of action of the resultant will run through the center of the circle, because the pressure is everywhere perpendicular to the surface and the center point is the only point laying on all perpendiculars.

Examples

Vertical

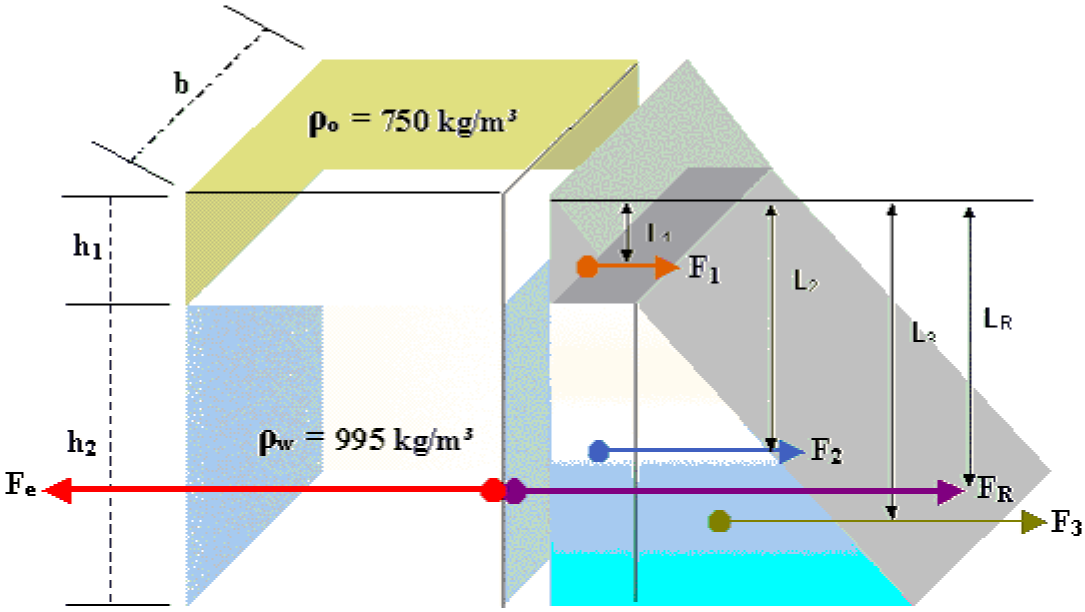
A tank with dimensions 2.8 x 3.0 x 4.0m is filled with water, with density 995 kg/m³, to the first three meters. On top of the water oil with a density of 750 kg/m³ floats one meter deep. Calculate the resultant hydrostatic force and the position of its line of action on the side of 2.8 x 4.0m high.

SOLUTION:

h_1 = 1m

h_2 = 3m

b = 2.8m



$F_1 = \bar{p}_{oil} A_1$

$= (0 + p_{oil} g h_1)/2 \times h_1 b$

$= (0 + (750)(9.81)(1))/2 \times 2.8$

$= 10.3$ kN at $L_1 = 2/3 \times 1 = 0.667$m

$F_2 = p_{oil} A_2$:➔ $A_2 = h_2 b$

$= p_{oil} g h_1 \times A_2$

$= (750 \times 9.81 \times 1) \times (2.8 \times 3)$

$= 61.8$ kN at $L_2 = 1 + 1.5 = 2.5$m

$F_3 = \bar{p}_{water} A_2$

$= A_2(0 + \rho w g h_2)/2$

$= (2.8 \times 3)(0 + (995 \times 9.81 \times 3)/2$

$= 122.99$ kN at $L_3 = 1 + 3(2/3) = 3$m

$F_R = F_1 + F_2 + F_3$

$= 10.3 + 61.8 + 122.99$

$= 195.09$ kN

$F_e = F_R$:➔ because SUM(M = 0) !!!

SUM(M = 0) : $F_e \times L_R = (F_1 \times L_1) + (F_2 \times L_2) + (F_3 \times L_3)$

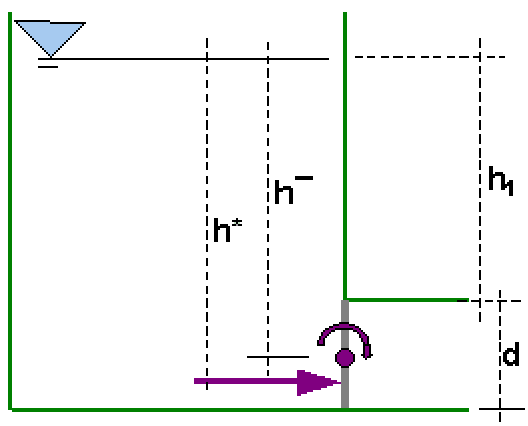
$195.09 \times L_R = (10.3 \times 0.667) + (61.8 \times 2.5) + (122.99 \times 3)$

$L_R = 2.72m$

Example to illustrate the independence of the moment $F_R e$ from the total height [h], with $e=h^* - \bar{h}$

A tank with a circular outlet is seen below, a butterfly valve **rotates about a horizontal shaft which runs through the centroid**, it closes vertically. Determine the horizontal force which the two bushes must withstand on either side of the shaft, the moment required to keep the valve closed, and explain why the moment required does not depend on the height/depth of the water level.

For the first computations the dimensions are as follows: water depth from free surface to top of butterfly valve is 2.5m **[h_1]**, the diameter of the circular butterfly valve is 1m **[d]**, the fluid is at rest with ρ=1000kg/m^3.



φ :➔ $\underline{\varphi=90°}$ thus sin90°=1

Direction:➔ perpendicular to the surface at **(90°-(φ=90°))=** *0° to the horizontal datum*

Magnitude:➔ **$F_R=\rho g \bar{h} A$** with **$A=\pi d^2/4$** and **$\bar{h}$ =2.5m + d/2** :➔ =*23,114 kN*

Thus each bush has to withstand *11,557kN*

Position:➔ $h^* = \bar{h} + (k^2 \sin^2\varphi)/\bar{h}$ = *3,028m*

Moment:➔ $F_R e$=*0,642kNm clockwise*

Derivation:➔ $M=F_R e$ with $e=(k^2\sin^2\varphi)/\bar{h}$ as from $e=h^*-\bar{h}$: ($\bar{h}$ - $\bar{h}$=0)

Leaving

$M=\rho g\,\bar{h}\,A\,(k^2\sin^2\varphi)/\bar{h}$

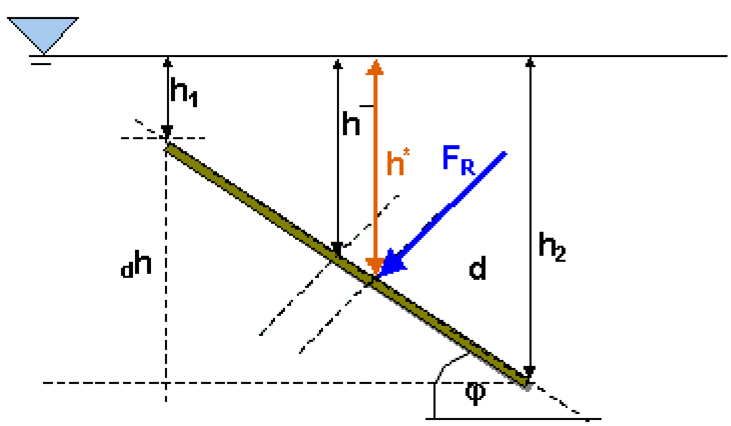
$=\rho g\,A\,k^2\sin^2\varphi$ with $Ak^2=I$

…the moment only depends on k^2 and φ

Inclined

A disc [$k^2=d^2/12$, $A=\pi d^2/4$, $I=\pi d^4/64$] with a diameter **d** of 1m is placed under water with ρ=1000 kg/m^3 so that the top edge is 0.6m and the bottom edge is 1.2m below the water surface. The fluid is at rest.

Determine the total hydrostatic forces to balance the disc, and draw up an outline sketch to show the different heights/depths, as well as the resultant forces.



φ :➔·· $_dh/d=\sin\varphi$ φ➔ *φ=36.87°*

Direction: **(90°-(φ=36.87°))**= *53.13° to the horizontal datum*

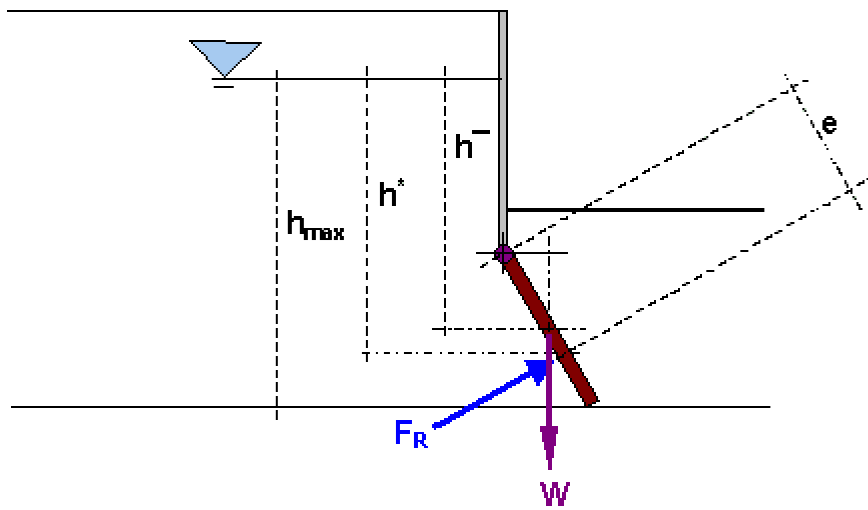
Magnitude➔ $F_R=\rho g\,\bar{h}\,A$ with $A=\pi d^2/4$, $\bar{h}=(_dh/2)+h_1$:➔ =*6.934 kN*

Centre of pressure→ $h^* = \bar{h} + (k2\sin 2j) / \bar{h}$ = *0.933m*

Example of indirect determination of a maximum water height by means of moment balancing

Imagine a rectangular cast iron sluice gate with the following dimensions: base [b=1.2m], height [d=0.9m] and a mass of 3000kg. With its weight, this sluice gate closes a canal outlet which has a maximum depth [h=2m]. The canal outlet is built with a vertical concrete wall, the wall is connected to the canal profile only at the sides. Under the wall the sluice gate is hinged to open in flow direction, the bottom of the sluice gate just rests on a concrete floor mounting into another canal. This cast iron sluice gate is inclined at 60° to the horizontal datum. [$g=9.81m/s^2$]

Determine the height [h_{max}] the water could just rise in the canal when it will start to force the sluice gate open. Such devices may be used to retain water in canals for irrigation purposes.



φ :→ $\varphi=60°$ thus sin60°=0.8660, cos60°=0.5

Compute Momentum$_{cl}$:→ $M_{cl}=mg(d/2)\cos\varphi$ (3000*9.81*0.45*0.5)= *6622Nm*

Direction:→ perpendicular to the surface at **(90°-(φ=60°))=** *30° to the horizontal datum*

Magnitude:→ $F_R=\rho g \bar{h} A$ with **A=bd** and $\bar{h} = h_{max} - (0.9 \sin 60°)/2$

$\bar{h} = (h_{max} - 0.39)$

F_R= 10595(h_{max} – 0.39) N

Position:➔ $h^* = \bar{h} + (k^2 \sin^2\varphi)/\bar{h}$ with $k^2 = d^2/12 = 0.0675m^2$

h^*= (0.0506/(h_{max} – 0.39)) + (h_{max} – 0.39)

Momentum$_{op}$:➔ $M_{op} = F_R\, e$

Excentricity:➔ **$e = [h^* - (h_{max} - d\sin\varphi)]/\sin\varphi$**

e=[(0.0506/(h_{max} – 0.39)) + (h_{max} – 0.39)-(h_{max}-0.78)]/ 0.8660

e=[(0.0506/(h_{max} – 0.39)) + 0.39] / 0.8660

Compute Momentum$_{op}$:➔·M_{op}=10595(h_{max} – 0.39)* [(0.0506/(h_{max} – 0.39)) + 0.39] / 0.8660

M_{op}= 12234(h_{max} – 0.39)* [(0.0506 + 0.39h_{max} – 0.152)/(h_{max} – 0.39)]

M_{op}= 12234[0.39h_{max} – 0.1014]

M_{op}= - 1240 + 4771h_{max}

Equilibrium:➔ $M_{cl} = M_{op}$

6622Nm = - 1240Nm + 4771h_{max}

for $\mathbf{h_{max}}$ = *1.648m*

General

A typical request for a water engineerinf technician to think across boarders of plain hydraulic engineering:

A commercial farmer along the Kuiseb River wants to place a round 9425 liter tank made of galvanized sheets on a tank stand made of steel pipes near the river to have basic facilities for irrigation. He further communicated that he needs a minimum static water pressure of 100kPa at the outlet (just above ground on the down pipe).

You estimated the setup (stand and empty tank) to have a total mass of 1075kg. The tank's dimensions are: diameter D=2m and height H=3m, the tank stand should have a square shaped base with a side length following the diameter of the tank. The area along the Kuiseb River is exposed to desert storms which can blow from any direction, thus the farmer has to expect wind speed maxima up to 35m/s. Such wind speed maxima may cause pressures up to 0,8kN/m^2. Work with a factor of 2 on a set of foundations along a side to include the worst case of gusts blowing across the diagonal of the tank stand. For such cases the cylindrical tank is regarded to have a plan surface area as if it would be a wall. The bearing capacity of the sandy area was found to be 150kN/m^2 at a depth <1m (safety factor included) and we assume the farmer will tie the tank to the stand so that it can not be blown of once it is empty.

You must now consult the farmer economical dimensions for the tank stand and the foundations to support the situation as outlined above. (assume g=10m/s^2)

Ignore horizontal shear and show the foundation's dimensions without the influence of wind and with the influence of wind, comment on your calculations. You may round up certain results to the nearest decimeter/meter or kN for easy computation but you must always be on the safe side and economic in your suggestion.

Produce an outline sketch to assist you respecting all load cases and dimensions.

----

minimum pressure head needed: 100kPa equiv. to 10m pressure head:

height of tank stand above ground=10m.

Dimensions of foundations without respecting wind influence:

Total weight: (9425 kg + 1075kg = 10500 kg)*g = 105kN (10,5 tons)

Regular Tank Stand has four legs, i.e. four foundations

Each foundation has to support around 27 kN. (26,25kN)

Sandy conditions (bearing capacity 150kN/m^2, depth <1m)

A_{min}=0,18m^2 chosen 0,25m^2 square shaped (50x50 ...cm for easy workability in the field)

Height of foundation block : tan60DEG=1,732 ……above 0,45m chosen 0,5m

Add weight of foundation (rel. density of concrete=2,4): 3 kN

Total Force to support per foundation = 30 kN

Total Pressure on sand: 30kN/0,25m^2 = 120kN/m^2 < 150kN/m^2 (20% safety, no wind respected)

Dimensions without wind influence:

width	length	height
50cm	50cm	50cm

Dimensions of foundation respecting wind influence:

Wind speed max=35m/s, q_w=0,8kN/m^2, c_f=1 (form factor)

$W_h=q_w*c_f*A_{tank}$ ….. A_{tank}=6m^2 ……W_h=4,8kN chosen 5kN

Lever above ground= 10m+1,5m=11,5m chosen 12m

$Momentum_{wind= 12*10=60kNm}$

Distance of foundation centers=2m (follow the diameter of the tank)

Forces to balance the momentum= ± 30kN (per 4 foundations along the sides)

Maximum vertical force on pre-dimensioned foundation: 60kN (incl: factor 2, diagonal gusts)

Minimum Force on pre-dimensioned foundation: 0kN

Re-dimension foundation (for p<150kN/m^2):

square shaped: Afoundation(wind+weight)>0,4m^2, side length 0,6325m <u>chosen 0,7m</u>

Height of foundation block : tan60DEG=1,732 ……hight>0,60m, <u>chosen 0,65m</u>

add.weight of foundation (rel. density of concrete=2,4): <u>close to 7,5kN</u>

ΣForces(max)=27kN+30kN+7,5kN=64,5kN (diagonal: 21,5 kN from wind influence)

Foundation area around 0,5m^2 ……. Pressure=130kN/m^2 <150kN/m^2

The tank stand will not tilt over during wind speeds up to 35m/s due to F_{min}= -30kN+34,5kN **>** 0

Dimensions with wind influence:

width	length	height
70cm	70cm	65cm

Hydrodynamics

Introduction

Fluid Dynamics or Hydrodynamics

This branch of fluid mechanics deals with the laws of **fluids in motion**; these laws are considerably more complex and, in spite of the greater practical importance of fluid dynamics, only a few basic ideas can be discussed here.

Interest in fluid dynamics dates from the earliest engineering application of fluid machines. Archimedes made an early contribution by his invention of the screw pump, the pushing action of which is similar to that of the corkscrewlike device in a meat grinder. Other hydraulic machines and devices were developed by the Romans, who not only used Archimedes' screw for irrigation and mine pumping but also built extensive aqueduct systems, some of which are still in use. The Roman architect and engineer Vitruvius first described the verticle waterwheel, a technology that revolutionized corn milling, during the 1st century BC.

Despite the early practical applications of fluid dynamics, little or no understanding of the basic theory existed, and development lagged accordingly. After Archimedes, more than 1800 years elapsed before the next significant scientific advance was made by the Italian mathematician and physicist Evangelista Torricelli, who invented the barometer in 1643, and formulated Torricelli's law, which related the efflux velocity of a liquid through an orifice in a vessel to the liquid height above it. The major spurt in the development of fluid mechanics had to await the formulation of Newton's laws of motion by the English mathematician and physicist Isaac Newton. These laws were applied to fluids first by the Swiss mathematician Leonhard Euler, who derived the basic equations for a frictionless, or inviscid, fluid.

Euler first recognized that dynamical laws for fluids can only be expressed in a relatively simple form if the fluid is assumed incompressible and ideal, that is, if the effects of friction or viscosity can be neglected. Because, however, this is never the case for real fluids in motion, the results of such an analysis can only serve as an estimate for those flows where viscous effects are small.

Fluid Flow Types

If any fluid is in motion, it can be classified according to its form and pattern of movement.

Background

Incompressible Flows

A. Incompressible and Inviscid, or Frictionless, Flows

These flows follow Bernoullis principlenamed after the Swiss mathematician and scientist Daniel Bernoulli. The principle states that the total mechanical energy of an incompressible and inviscid flow is constant along a streamline. Streamlines are imaginary flow lines that are always parallel to the local direction of the flow, and that for steady flow are also the lines followed by individual fluid particles. Bernoulli's principle leads to an interrelationship between pressure effects, velocity effects, and gravity effects, and indicates that the velocity increases as the pressure decreases. This principle is important in nozzle design and in flow measurements.

Viscous Flows

B. Viscous Flows, Laminar and Turbulent Motion

The first carefully documented friction experiments in low-speed pipe flow were carried out independently in 1839 by the French physiologist Jean Leonard Marie Poiseuille, who was interested in the characteristics of blood flow, and in 1840 by the German hydraulic engineer Gotthilf Heinrich Ludwig Hagen. An attempt to include the effects of viscosity into the mathematical equations was made first in 1827 by the French engineer Claude Louis Marie Navier, and independently by the British mathematician Sir George Gabriel Stokes, who in 1845 perfected the basic equations for viscous incompressible fluids. These are now known as the Navier-Stokes equations, and they are so complex that they can be applied only to simple flows. One such flow is that of a real fluid through a straight pipe. Here Bernoulli's principle is not applicable because part of the total mechanical energy is dissipated as a result of viscous friction, resulting in a pressure drop along the pipe. The equations suggest that this pressure drop for a given pipe and a given fluid should be linear with the flow velocity. Experiments first conducted near the middle of the 19th century showed that this was only true for low velocities; at higher velocities, the pressure drop was more nearly proportional to the square of the velocity. This problem was not resolved until 1883 when the British engineer Osborne Reynolds showed the existence of two types of viscous flows in pipes. At low velocities the fluid particles follow the streamlines (laminar flow) and results match the analytical prediction. At higher velocities the flow breaks up into a fluctuating velocity pattern or eddies (turbulent flow) in a form that cannot be fully predicted even today. Reynolds also established that the transition from laminar to turbulent flow was a function of a single parameter that has since become known as the Reynolds number. If the Reynolds number, which is the product of velocity, fluid density, and pipe diameter, divided by the fluid viscosity, is less than 2100, the pipe flow will always be laminar; at higher values it will normally be turbulent. The concept of a Reynolds number is basic to much of modern fluid mechanics.

Turbulent flows cannot be evaluated solely from computed predictions and depend on a mixture of experimental data and mathematical models for their analysis, with much of modern fluid-mechanics research still being devoted to better formulations of turbulence. The transitional nature from laminar to turbulent flows and the complexity of the turbulent flow can be observed as cigarette smoke rises into very still air. At first it rises in a laminar streamline motion but after some distance it becomes unstable and breaks up into an intertwining eddy pattern.

Boundary Layer Flows

C. Boundary Layer Flows

Before about 1860 the engineering interest in fluid mechanics was limited almost entirely to water flows. The development of the chemical industry during the latter part of the 19th century directed attention to other liquids and to gases. Interest in aerodynamics began with the studies of the German aeronautical engineer Otto Lilienthal in the last decade of the 19th century and saw major advances following the first successful powered flight by the American inventors Orville and Wilbur Wright in 1903.

The complexity of viscous flows, especially turbulent flows, severely restricted progress in fluid dynamics until the German engineer Ludwig Prandtl recognized in 1904 that many flows could be divided into two principal regions. The region close to the surface consists of a thin boundary layer where the viscous effects are concentrated and where the mathematical model can be greatly simplified. Outside the boundary layer viscous effects can be disregarded and the simpler mathematical equations for inviscid flows can be used. The boundary-layer theory has made possible much of the development of modern aircraft wings and the design of gas turbines and compressors. The boundary-layer model not only permitted a much simplified formulation of the Navier-Stokes equations in the region close to the body surface but also led to further developments of the flow of inviscid fluids that can be applied outside the boundary layer. Much of the modern development of fluid mechanics was made possible by the boundary-layer concept and it has been carried out by such key contributors as the Hungarian-born American aeronautical engineer Theodore von Kármán, and the German mathematician Richard von Mises, by the British physicist and meteorologist Sir Geoffrey Ingram Taylor.

Compressible Flows

D. Compressible Flows

Interest in compressible flows started with the development of steam turbines by the British inventor Charles Algernon Parsons, and the Swedish engineer Carl Gustaf Patrik de Laval during the 1880s. Here high-speed flow of steam within flow passages was first encountered and the need for efficient turbine design led to improved compressible flow analyses. Modern advances, however, had to wait for the stimulus of successful gas turbine and jet engine development in the 1930s. The early interest in high-speed flows over surfaces arose in the study of ballistics, for which an understanding of the motion of projectiles was needed. Major developments started near the end of the 19th century, involving Prandtl and his students, among others, and increased after the introduction of high-speed aircraft and rockets in World War II.

One of the basic principles of compressible flows is that the density of a gas changes when the gas is subjected to large velocity and pressure changes. At the same time its temperature also changes, leading to more complex means of analysis. The flow behavior of a compressible gas depends on whether the flow velocity is smaller or greater than the velocity of sound. The velocity of sound is the name given to the propagation velocity of a very small disturbance, or pressure wave, within the fluid. For a gas it is proportional to the square root of the absolute temperature. For instance, air at 20° C, or 293° on the Kelvin, or absolute, scale (68° F), has a sound velocity of 344.65 m per sec. If the flow velocity is less than the sound velocity (subsonic flow), pressure waves can be transmitted throughout the whole fluid to adjust the flow that rushes toward an object. Thus the subsonic flow approaching an airplane wing will adjust itself some distance upstream to flow smoothly over the surface. In supersonic flow, pressure waves cannot travel upstream to readjust the flow. As a result, the air rushing toward a wing in supersonic flight will not be prepared for the impending disturbance the wing will cause. Instead, it has to redirect very suddenly in the proximity of the wing, where a sharp compression or shock is coupled with the redirection. The noise associated with this sudden shock causes the sonic boom of aircraft flying at supersonic speeds. Compressible flows are often identified by the Mach number, which is the ratio of the flow velocity divided by the sound velocity. Supersonic flows therefore have a Mach number greater than 1.

Streamline

...is an imaginary line which can be drawn through the flow region so that the **velocity vectors** of the flow are **tangential** to the streamline at each point of the flow.

The tangent at any point on the streamline indicates the direction of the flow at that point. The velocity of the flow does not have to be constant, it can vary from from point to point and also with time.

If a number of streamlines are drawn of a stream cross-section, they will form a **stream tube**. This stream tube is an imaginary tube and there is no flow through the walls of the tube. This tube's shape can vary from time to time and from place to place. It forms a useful principle, as it can be used to isolate a small section of the flow for further analysis.

Simulation

For a simulation of streamlines, smokelines, velocity vectors and pressure field changes on various objects submerged in a flow stream see: Simulation(IRRO)

Further Information supplied by **John S. Denker**, who wrote some very enjoyable pages on the principles of aerodynamics. (See Table of ContentsOnline)

1-D Flow

...we consider the changes in flow only in one dimension (x), the conditions in the other two dimensions (y, z) are assumed constant.

If a fluid flows through a pipe we can assume the flow velocity as constant throughout the pipe, at the pipe walls as at the center of the pipe, considering this velocity as **average velocity**(**v_{av}**). The velocity as the pressure may actually change along the length of the pipe. The pipes path in space is the only dimension along which conditions may change.

2-D Flow

...is where we assume the flow takes place **in parallel two dimensional planes (x, y)** and on **identical paths** in each of these planes.

If a fluid flows over a weir (dam wall), we assume that the flow pattern of the fluid over the weir is the same at any point over the weir.

Cross-section$_1$ = Cross-section$_2$

3-D Flow

...is where the flow and the **conditions of flow (p, T, ...)**are described in all **three dimensions (x, y, z)**. A modern specialization field of engineering/maths/physics is **computational fluid dynamics**, in which computers/software are used to compute the conditions in the flow field. These computations may eliminate much of the experimental work that was previously required when researching the behaviour of flow in non linear cases.

Laminar Flow

...is when flow is so slow that **no mixing of the fluid takes place (Re<2300)**, as if the fluid consists of tiny thin layers sliding parallel over each other.

Newtons law of viscosity is applicable here ($\tau=\upsilon(d\mathbf{v}/dy)$).

Turbulent Flow

...takes place **when regions of a fluid move in an unorderly way on a flow path**, not in parallel layers, but intermixing the layers.

Because there is continuously a large interchange of momentum between the molecules in different layers of the fluid, the layers can no longer move as easily over each other as in the case of laminar flow, resulting in a higher effective viscosity.

..two liquids mixing

Ideal Flow

...is described as the **flow of a fluid which has no viscosity**, therefore no shear stress in the fluid, only normal pressures. This is just an assumption to simplify certain flow situations, because all real fluids do have viscosity, yet it might be negligibly small in some cases.

Steady Flow

...takes place, when flow conditions do not change with time (t). In such case the velocity, pressure, density and temperature remain constant.

($d\mathbf{v}/d\mathbf{t}$, $d\mathbf{p}/d\mathbf{t}$,$d\rho/d\mathbf{t}$,$d\mathbf{T}/d\mathbf{t} = 0$)

For unsteady flow the above in brackets are no longer equal to zero

Control Volume Principles

The href="#Conservation of Mass"conservation of mass

Newton's laws of motion must be applicable to every fluid particle

The first and second laws of thermodynamics must be applicable

Boundary conditions must be obeyed to such as:

v=0 at stationary walls

the fluid can not pass through walls

When the conditions of a moving fluid are changed in a certain region, it is convenient to image a box around this region, and then monitor the flow through the boundaries of that box.

This flow includes:

fluid flows

forces

energy (heat, electrical or mechanical etc.)

As nothing can continue to build up in the box, it follows, that what enters the box has to leave it again. **[In=Out]**

Such box is called a **controll volume** and the sides are called the**controll surfaces**(see:boundary conditions)

Conservation of mass

Mass Flow Rate

If a fluid flows through a controll volume, the **mass flow rate** entering the box has to be equal to the mass flow rate leaving the box.

$m_{\text{flow rate (in)}} = m_{\text{flow rate (out)}}$ = $\dot{m}_{in} = \dot{m}_{out}$

which is:

$\rho_{in} A_{in} V_{in} = \rho_{out} A_{out} V_{out}$

with the dimensions:

$(kg/m^3)(m^2)(m/s)$ = [kg/s]

A is the cross sectional area of the flow perpendicular to the flow direction, and **V** is the average velocity of the flow at the specific position perpendicular to **A**.

(Note: The real velocity pattern of the flow differs from point to point. Near the wall of a pipe the velocity is lower than in the centre of a pipe. This influence will be discussed under friction losses in pipes see volumetric flow ratetoo.)

Volumetric Flow Rate

Now, if the incoming and out going density is the same, the fluid is **incompressible** (water, oil, etc.).

Then, and only then the following is valid:

$A_{in} V_{in} = A_{out} V_{out}$ >>>with the dimension: **$(m^2)(m/s)$ = [m^3/s]**

The product **AV** is called **Q**, the **volumetric flow rate**.

Again, **if the density remains constant, the volumetric flow rate remains constant** too (in/out).

This equation is also known as the **continuity equation**, $Q_{in} = Q_{out}$

where we have to compute a mass balance !!

In multiplying Q with ρ ,we get the mass flow rate $\dot{m}$.

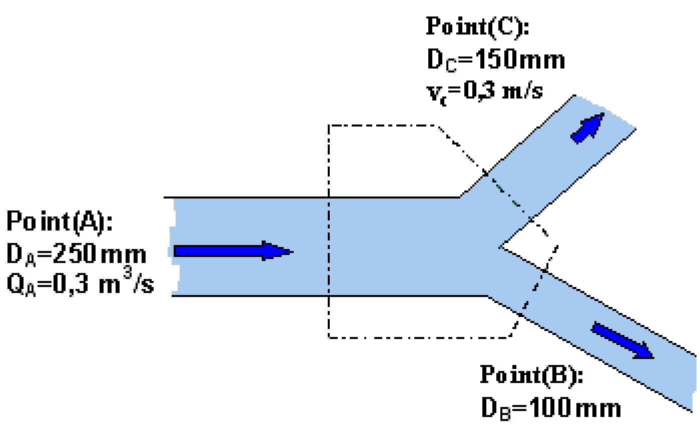
Examples

Examples to show the handling of volumetric flow rate, continuity equation and mass flow rate

Flow Splitting in Pipes

Consider the following setup: A pipe with diameter 250mm mounts into two pipes with diameters 150mm and 100mm respectively. At point(A) the volumetric flow rate is known with 0,3 cubic meters per second, at point(C) a valve is installed and does not close propperly, the flow velocity is known with 0,3 meters per second. We are interested in the flow velocity **v** in points(A, B) and in the two outgoing volumetric flow rates (Q_B, Q_C). Also compute the mass flow rates at (A, B, C).

The density of the fluid is constant with 1000 kg/m^{3}at all three points. **(Note: We draw a controll box around the points of interest (A, B, C).)**



We apply the **continuity equation**:

$$Q = A_{in} V_{in} = SUM(A_{out(i)} V_{out(i)})$$

with Q_A= 0,3 m^3/s =$\underline{Q_B+Q_C}$

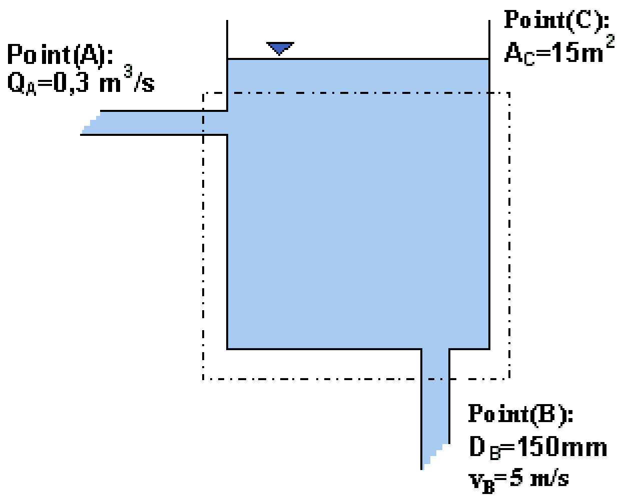
$= A_{(B)} V_{(B)} + A_{(C)} V_{(C)}$ ..... 37,5 m/s for $V_{(B)}$

Now we can compute the volumetric flow rates (B, C) ..... Q_C = 0,01 and Q_B = 0,29 in **[m^3/s]**

...and the average velocity for point (A) ... $V_{(A)}$ = 6,1 **[m/s]**

Flow Splitting in Tank

Water flows into an open tank at **piont**(A) at a volumetric flow rate **Q**A= 0,3 m³/s, and flows out at the bottom of that tank again at **point**(B) through a pipe with 150mm diameter at an average velocity of 5 m/s for **V**(B). The area of the water surface at **point**(C) is given with 15 square meters. We are interested in the water level at **point**(C), the speed it will fall or rise. The density of the fluid is constant with 1000 kg/m³at all three points. **(Note: We draw a controll box around the points of interest (A, B, C).)**



We apply the **continuity equation**:

Q =AinVin = SUM(Aout(i)Vout(i))

with **Q**A= 0,3 m³/s =**QB+QC**

= **A**(B)**V**(B)**+ A**(C)**V**(C) ..... +0,01 m/s for **V**(C), the water level will rise.

Return to:

>Mass Flow Rate< >Volumetric Flow Rate<

Compressing Air

A compressor takes in air with a density of 1,23 kg/m³ at a volumetric flow rate 50 litres/second. The air is compressed when it leaves the compressor through a 25mm diameter pipe at a velocity of 28 m/s. We are interested in the final density of the compressed air.

We apply: **m**flow rate (in)=**m**flow rate (out) = $\dot{m}_{in} = \dot{m}_{out}$ ..... ρ_{in}**Q**in= ρ_{out}**A**out**V**out ..... 4,47 kg/m^3 for $\rho_{air(out)}$

Conservation of momentum

To recall from High School Sciences:

When an object or particle is moving, the **product of its mass and its velocity** is called the **momentum = (mv)**.

If now a force **F** acts against an object or particle causing the velocity to change, then this is called an **impulse** and is defined as the product of the force **F** times the time duration **t**. This impulse causes a change in momentum.

in formula: **Ft = m(v_2- v_1)** with **F= m(v_2- v_1)/t** with **m/t=** $\dot{m}$ following Newton's second law (F=ma =m(v_2 - v_1)/t). **[(mkg/s^2)N]**

Law of Conservation of Momentum

Now suppose, the object is a quantity of fluid with a mass **m**that passes through a nozzle or bend (control volume) during the time **t**, then (F=ma =m(v_2 - v_1)/t) is still valid. If the fluid is flowing at a constant flow rate, then **m/t=** $\dot{m}$ is the mass flow rate discussed earlier under conservation of mass.

(Note: The force Facted upon the fluid and accelerated/retarded it. The force can originate from the nozzle walls, water pressure, or gravity. Often effects of gravity are so small, that they are disregarded and only the effect of the sum of the other forces is taken into consideration.)

the above note in formula: **F_x=m(v_{2x} - v_{1x})**/t with **m/t =** $\dot{m}$ Because **forces and velocities are vectors**, the direction has to be included, therefore the index **x** (subscript) is used to indicate the direction.

This leads us to the **law of conservation of momentum**:

The sum of all external forces in a definite direction (x, y, z) which act upon a fluid in a control volume

=

the product of the mass flow rate through the control volume and the velocity change in the same direction as the force.

(F_x=m(v_{2x} -v_{1x})/t with **m/t =** $\dot{m}$ **)**

This law forms the base of the theory about forces and stresses in pipe systems, the action of turbine rotors, pump impellers, etc.

where we must balance the different vectors Fand v.

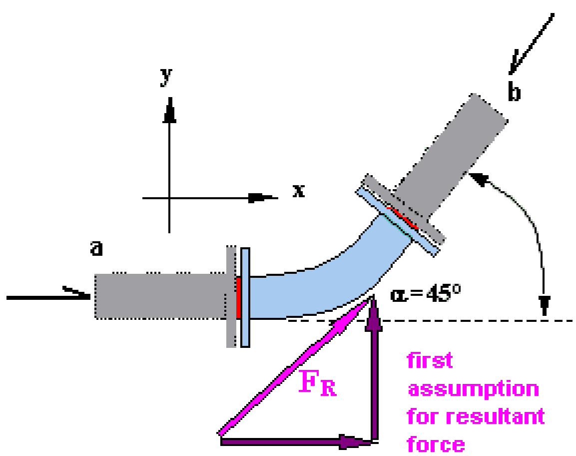
Examples

Pipes&Bend

Example to demonstrate the law of conservation of momentum

Consider the following plan view of a pipe and bend setup. The internal pipe and bend diameter is 0,25m, Q_a = 0,25 m^3/s= Q_b. The gauge pressurepat both the inlet and the outlet of the bend is 200 kPa, the density of a fluid is constant with 1000 kg/m^3. We are interested in the magnitude and direction of the resultant force which the pipe flanges and the pipes must exert to keep the bend in position. Our control volume is the bend.

(Note: To solve this type of problem, the pressure (see also hydrostatic pressure) and the velocities are required at the inlet and outlet. The velocities and pressures are not always readily available and must be determined with the aid of: conservation of mass, and/or Bernoulli's equation, before the momentum equations can be set up.)



Consider the forces across the control volume as shown in the sketch above.

The pressure forces from the adjacent water in the pipe which are: p_1A_1 and p_2A_2, in our case they are equal, because the diameter of both pipes at the inlet to the bend and at the outlet of the bend are the same.

The force which the wall of the bend exerts upon the fluid is F_R , assume it in the direction as shown in the sketch with its two component forces in (x, y) direction, it might be corrected through computing the resultant force ("sign reversal" will then show opposite direction).

The weight of the fluid in the bend. As we are in the plan view, this acts vertically in (z) direction and will therefore not influence the two component forces in (x, y) direction, so we ignore it.

Conservation of energy

To recall from High School Sciences:

In fluid mechanics a very important principle is employed: **the fact that energy can not be destroyed, lost or gained!**

Energy can only be converted to another form.

This principle is also known as **the first law of thermodynamics**.

The types of energy of concern to water engineering are mainly the **mechanical energy of a moving stream or fluid particle**, and the**thermal energy**which results from friction between fluid molecules and on the interface fluid/wall of a pipe.

The Energy Types

We consider a moving fluid with a mass flow rate $\dot{m}$ and a velocity **v** that flows through a control volume where a certain amount of energy is "lost" or "gained" at a certain rate. In such case energy can be added by a pump, by the thermosyphonic cycle in a solar water heater, or it can be transferred ("lost") to a turbine for hydropower generation, or simply heat up the fluid.

The flow of this moving fluid contains at least three types of **mechanical energy**:

Pressure Energy: ...the static pressure**p**of the fluid multiplied by the cross sectional area**A**of the flow stream (this gives us the pressure force) times the velocity of that flow stream **V**. *In formula:* $P_p = pAv = pQ = \dot{m} \times p/\rho$ .....(with P_p = pressure energy of a flow stream and $m/t = \dot{m}$)

Kinetic Energy:... the flow stream's velocity implies that there is kinetic energy available at an energy flow rate of

In formula: $P_{ke} = \dot{m}v^2/2 = \rho Q v^2/2 = \rho A v^3/2$.....(with P_{ke} = kinetic energy of a flow stream and $m/t = \dot{m}$)

Potential Energy: ...the higher the stream above a reference level, the more work the stream can do! It can be piped down causing the pressure to increase (see:hydrostatic forces, and hydro static pressure) and then passed through a nozzle for the average flow velocity to increase, to then drive a turbine.

If a stream is **z** meters above a reference level the potential energy flow rate is:

In formula: $P_{pe} = \dot{m}gz$ .....(withP_{pe} potential energy of the flow stream and $m/t = \dot{m}$)

The flow stream also contains other energy types:

Thermal Energy: ...in form of internal energy **u** [J/kg], the sun has heated it up for example, or we get heat losses

In formula: $P_{te} = \dot{m}u$ .....(withP_{te} thermal energy of the flow stream and $m/t = \dot{m}$)

The **Work**which is done **on** a fluid passing through a pump, or the work done **by**a fluid passing through a turbine is designated P_w.

Return to:

>The Energy Equation (General Form) [W]< >The Energy Equation (Pressure Head) [m]<
>Bernoulli's Equation<

The Energy Equation (General Form) [W]

Now we have to apply the conservation of energy to a steady flow stream, flowing through a control volume

(Note: to conserve energy means, that no energy can be "lost" while passing through the control volume):

Let us set up two tables to give us an overview! (Refer back to: Energy Types)

In formula:

$P_{in} = P_{out}$																		
Energy into the control volume (i)									=	Energy out of the control volume (o)								
P_p	+	P_{ke}	+	P_{pe}	+	P_{te}	+	P_w	=	P_p	+	P_{ke}	+	P_{pe}	+	P_{te}	+	
$\frac{pm}{\rho t}$	+	$\frac{mv^2}{2t}$	+	$\underline{mgzt}$	+	$\underline{mut}$	+	P_w	=	$\frac{pm}{\rho t}$	+	$\frac{mv^2}{2t}$	+	$\underline{mgzt}$	+	$\underline{mut}$	+	
pAv	+	$\frac{\rho Qv^2}{2}$	+	$\underline{mgzt}$	+	$\underline{mut}$	+	P_w	=	pAv	+	$\frac{\rho Qv^2}{2}$	+	$\underline{mgzt}$	+	$\underline{mut}$	+	
pQ	+	$\frac{\rho Av^3}{2}$	+	$\underline{mgzt}$	+	$\underline{mut}$	+	P_w	=	pQ	+	$\frac{\rho Av^3}{2}$	+	$\underline{mgzt}$	+	$\underline{mut}$	+	

Each term represents power, or an energy flow rate with the dimension [Nm/s, W]

Return to:

>Bernoulli's Equation< >The Energy Equation (Pressure Head) [m]<

The Energy Equation (Pressure Head) [m]

(Note: If the fluid is incompressible then $\rho_{in} = \rho_{out} = \rho$

and we can devide all terms in the above table by the weight flow rate ($\dot{m}g$) = ρQg to simplify, (refer back to: Mass Flow Rate)

then the terms represent (Power/weight flow rate) an all component pressure head in meters fluid. [m])

In formula:

The relationship □ P_{in} = □ P_{out} has to be divided by ρQg																	
Total pressure head into the control volume (i)									=	Total pressure head out of the control volume (o)							
$\frac{P_p}{\rho Qg}$	+	$\frac{P_{ke}}{\rho Qg}$	+	$\frac{P_{pe}}{\rho Qg}$	+	$\frac{P_{te}}{\rho Qg}$	+	$\frac{P_w}{\rho Qg}$	=	$\frac{P_p}{\rho Qg}$	+	$\frac{P_{ke}}{\rho Qg}$	+	$\frac{P_{pe}}{\rho Qg}$	+	$\frac{P_{te}}{\rho Qg}$	+
$\frac{p}{\rho g}$	+	$v^2 2g$	+	z	+	u/g	+	$P_w \rho Q g$	=	$\frac{p}{\rho g}$	+	$v^2 2g$	+	z	+	u/g	+

Notes:

The terms $(u/g)_o$ and $(u/g)_i$ are subtracted to obtain the loss in pressure head h_l.

This difference constitutes the internal energy balance (thermal energy) and shows the magnitude of energy converted into thermal energy.

An increase in internal, thermal energy is not desired in water engineering, because a higher heat grade is of no use, and constitutes a loss.

The term ($\tilde{P}_w \rho Qg$) is associated with pump power and it is abbreviated to h_p which is the total pressure head supplied by the pump to the flow stream.

The conservation of energy has so been converted to an equation of pressure head terms and it is known as the energy equation

energy equation																	
all pressure heads into the control volume (i)"gains"									=	all pressure heads out of the control volume (o)"l							
$p_i \rho g$	+	$v_i^2 2g$	+	z_i	+			h_p	=	$p_o \rho g$	+	$v_o^2 2g$	+	z_o	+	h_l	

Notes:

The term ($p/\rho g$) is the static pressure head, ($v^2/2g$) is the dynamic pressure head, (z) is the potential pressure head and the sum of all three is the total pressure head.

If there is no pump involved in the control volume, the term (h_p) = 0

The term (h_l) represents the losses that occur due to friction or viscous losses (internal friction), or due to turbulence (sudden changes in flow direction) it can also be expressed in terms of kinetic energy loss ($v^2/2g$) or pressure energy loss ($p/\rho g$).

Examples

Tank Discharge (Suction)

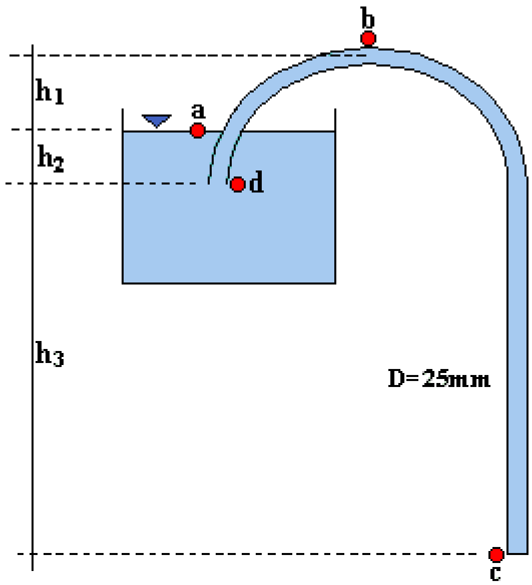
Tank Discharge (Suction)

Consider the following tank an pipe arrangement. (A similar setup could be a 200 litre drum and a hose, the drum standing at the back of a bakkie, and you want to fuel up the bakkie's tank.) The kinetic energy losses in the pipe have been found with 0,25($v^2/2g$) per meter length of pipe, where v is the average velocity in the pipe. The respective pipe lengths are (d to b = 5m) and (b to c = 15m), the diameter of the pipe is D=25mm. The heights are: h_1=2m, h_2=3m and h_3=7m. The fluid is water with a density of (ρ_{water}=1000kg/m^3) .

We are interested in:

...the discharge (volumetric flow rate through the pipe)

...the pressure at point (b)

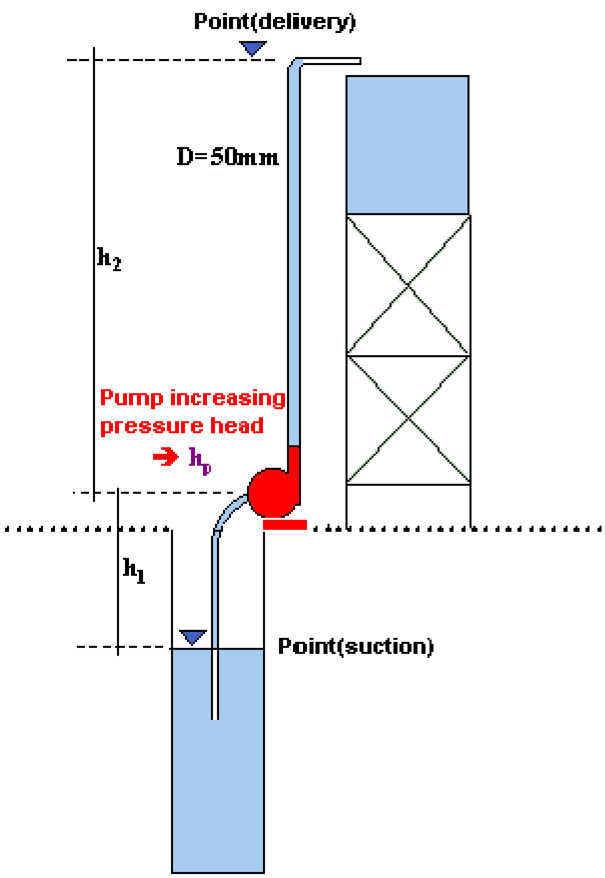


Pump Dimensioning

Example of dimensioning a pump: Water is worked on

Water (1000 kg/m^3)must be pumped from a well h_1=2m deep to a tank h_2=8m high, the well is recharging instantly, so we neglect the drawdown. The pipe diameters are 50mm. The total pressure head loss as a result of friction (pressure head loss in the single pipe pieces and the different bends) was found to be h_l=2m. The efficiency of the pump is ε =70%.

1. Determine the power the pump needs to lift a minimum rate of Q=10l/s.
2. Further determine the ratio of kinetic to potential energy in %.



Bernoulli Equation

When a **steady flow** along a streamline occurs (dv/dt)=0, with an **incompressible fluid** (ρ)=const)which has**no viscosity**(υ=0), and there is **no energy added or removed** from the fluid then the following can be deduced:

$(p/\rho g) + v^2/2g + z = \text{constant}$

In other words, the total energy per unit weight moving along such a streamline remains constant.

This equation is known as **Bernoulli's Equation**, and because of its similarity to the energy equation, the energy equation itself is often considered as an extension of, and in fact referred to as Bernoulli's equation. **(Note: follow the above link to review the energy conservation law)**

Or we can formulate Bernoulli's equation for a control volume (**i** conditions for flow stream in, **o**conditions for flow stream out):

$(p_i/\rho g) + v_i^2/2g + z_i = (p_o/\rho g) + v_o^2/2g + z_o$

How to handle "Bernoulli"

Notes on the application of Bernoulli's equation:

Always make an outline sketch and indicate all heights, velocities, pressures, forces and other useful information.

Choose two boundaries for the control volume (you have two sides of an equation) and collect all information to compute all but one parameter.

Set up the Bernoulli equation in the direction of the flow. Energy added to the control volume must be placed on the left hand side, and energy removed from the control volume must appear on the right hand side.

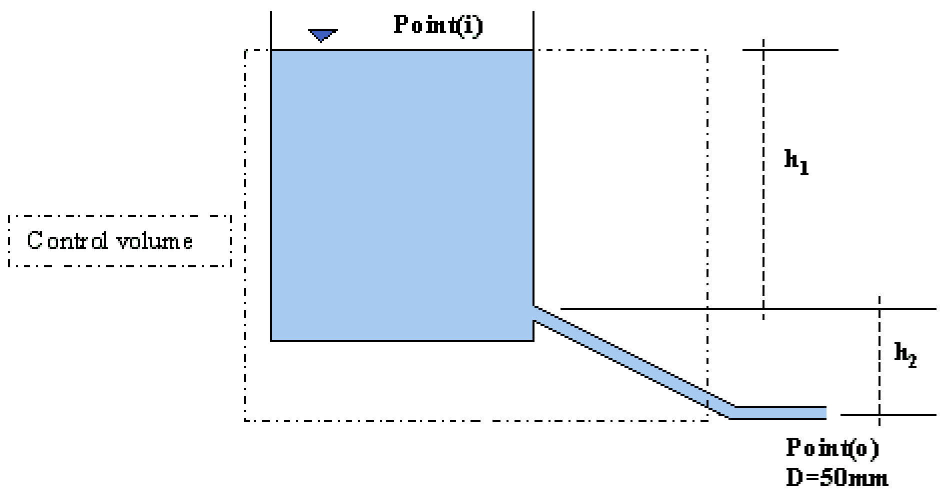
If there are two or more unknown parameters, one must first determine a relation between them with the continuity equation.

Examples

Tank Discharge

Tank Discharge

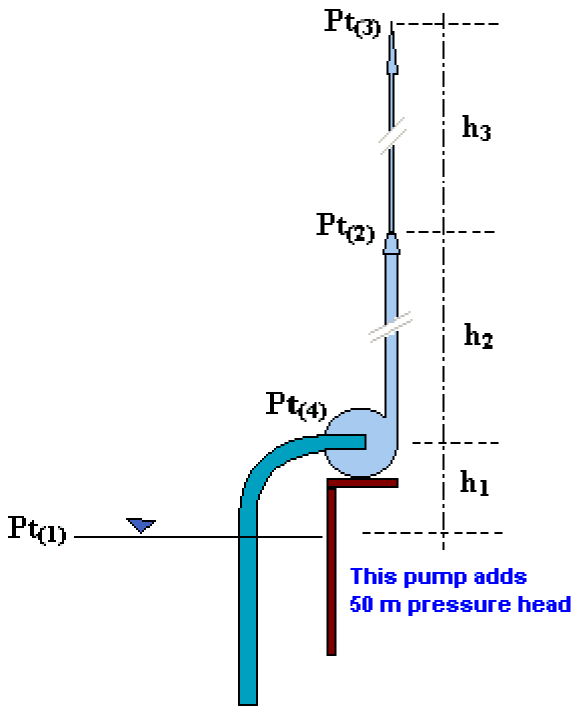
Consider an open tank to which a pipe is connected as in the outline sketch below. The water level in the tank is h_1 (2m) higher than the outlet of the tank, and the outlet of the pipe is h_2 (1m) lower than the outlet of the tank. The diameter of the pipe is given with D_{pipe} (50mm). Ignore all friction losses and compute the flow rate Q at the outlet of the pipe. ($\rho_{water}=1000kg/m^3$)



"Bernoulli-Mix"

"Pump&Nozzle"

Water is pumped from a well h_1 lower than the pump to a nozzle which is h_2 higher than the pump and it is discharged vertically upwards up to h_3. All pipes have a diameter of D_1=100mm and the nozzle diameter is D_2=25mm. The pump develops a total pressure head of h_p=50m, i.o.w. the pump adds h_p of pressure head to the water. We ignore all losses due to turbulence, friction in the pipe etc. (h_1 = 2,5m, h_2 = 17,5m, h_3= $z_{o(2)}$, ρ_{water}=1000kg/m^3)



We are interested in:

1. ...the velocity of the water when it just leaves the nozzle
2. ...the height the water will reach above the nozzle
3. ...the absolute pressure at the inlet of the pump if atmospheric pressure is 95 kPa.

Water is pumped from a well *(point"1")* h_1 lower than the pump*(point"4")*to a nozzle *(point"2")* which is h_2 higher than the pump and it is discharged vertically upwards up to h_3*(point"3")*. The well is re-charging instantly.

All pipes have a diameter of 100mm and the nozzle diameter is 25mm. The pump develops a total pressure head of 50m.

Pressure head losses due to turbulence, friction in the pipes and fittings amount to 2,5m.

The parameters relevant are:

(h_1= 2,5m, h_2= 10m, ρ_{water}=1000kg/m^3, g=10m/s^2)

Please compute the following parameters and clearly state the boundary conditions.

We are interested in:

1)...the velocity of the water when it just leaves the nozzle

2)...the height the water will reach above the nozzle

3)...the absolute pressure at the inlet of the pump if atmospheric pressure is 95 kPa.

Hydropower

...is the rate at which work is done by or on a fluid.

The energy equation (Bernoulli) in its basic form in conjunction with the control volume principle can be employed to determine the power in [Nm/s, W].

(Note: quite often some of the energy terms ar negligibly small, so simplifying the equation.)

With long pipelines or pipe nets with a number of valves and fittings, the friction losses can be remarkably high so a lot of power is expended in just overcoming the friction.

In general the power available in a flow stream of a fluid (ρ with relation to a reference level (horizontal datum) is expressed as:

$P=\rho gQ((p/\rho g) + v^2/2g + z)$ in [Nm/s, W]

With ρgQ = weight flow rate = mass flow rate times g

and $(p/\rho g) + v^2/2g + z$ is Bernoulli's equation

Turbine Equation

If a turbine has to transform the hydraulic power of a flow stream into **electric power**, the **efficiency of the conversion by the machine and its parts** must still be respected, as well as the **corresponding pipe losses**.

In such case, the general "**turbine equation**" is emloyed:

$$P_T=\rho gQh_n\varepsilon_T\varepsilon_G\varepsilon_{Tr}\varepsilon_{Coup}$$

with $h_n=h_{stph} - h_{thl}$ (h_{stph} = static pressure head(z), h_{thl} = total head loss: friction losses in pipes, bends, fittings etc. **[m]**)

P_T = Turbine power output **[kW]**

ρ = density of fluid **[t/m³]** **(Note: the similarity of t/m³ to the realtive density S)**

Q = volumetric flow rate **[m^3/s]**

h_n = net pressure head **[m]**

ε_T = Turbine efficiency (70-90%)

ε_G = Generator efficiency (85-95%)

ε_{Tr} = Transformer efficiency (95-99%)

ε_{Coup} = Coupling efficiency (96-99%)

The "rule of thumb of power output" for a turbine: $P_T=8h_nQ$

Examples

Simple Flow Stream

Example to demonstrate the calculation of hydraulic power in a simple flow stream

Imagine a flow stream with a volumetric flow rate of 1 m^3/s to drop down a 10m waterfall. The fluid density is 1000 kg/m^3. Determine the power available in the flow stream at the bottom of the waterfall.

Employ the above equation for hydraulic power: $P=\rho gQ((p/\rho g) + v^2/2g + z)$ in [Nm/s, W]

First discuss the three terms of the Bernoulli component of the equation.

The pressure difference between the top (10m) and the bottom (0m) of the waterfall is zero due to atmospheric pressure only. The term $p/\rho g$ will thus be ignored.

The velocity difference between the top (10m) and the bottom (0m) of the waterfall is negligibly small, thus the term $v^2/2g$ is set to zero as well.

Leaves $P=\rho gQ(+ z)$ in [Nm/s, W] ..... 98.100 W = 98,1 kW (theoretically available power, will not be available once the flow stream is channelled in a pipeline to brought to a turbine for power transformation, see general turbine equation below)

This power is "lost" to the flow stream in form of heat energy, the flow stream simply gets heated up.

Turbine: Water does work

Example to demonstrate the use of the general turbine equation: Water does work

Imagine a flow stream (water) channelled through a pipeline from the top of a hill down to a turbine 200m below the hill. The volumetric flow rate is $1 m^3/s$. The total pressure head loss due to pipe friction and loss in fittings, nozzles etc. was calculated to 50m. The fluid density is 1000 kg/m^3.

We are interested in

1. ... the theoretically available power of the flow stream
2. ... the power available at the turbine
3. ... the transmission efficiency of the pipeline
4. ... the net power output of the turbine if η_T = Turbine efficiency (90%), η_G = Generator efficiency (93%), η_{Tr} = Transformer efficiency (98%), η_{Coup} = Coupling efficiency (97%)
5. ... overall efficiency (the net power output of the turbine in relation to the theoretically available power)
6. ... the net power output according to the "rule of thumb of power output"

We employ the general "turbine equation": $P_T = \rho g Q h_n \varepsilon_T \varepsilon_G \varepsilon_{Tr} \varepsilon_{Coup}$

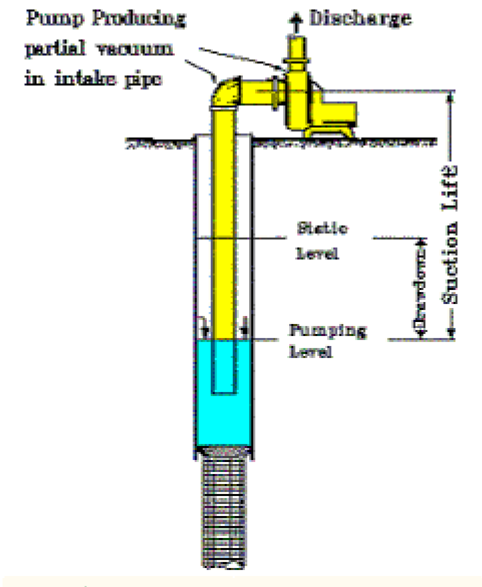
1. $P_{theor} = \rho g Q (+ z)$ in [Nm/s, W] ..... 1.962 kW
2. $P_{T(at)} = \rho g Q h_n$ in [Nm/s, W] ..... 1.472 kW (Note: the total pressure head loss decreases the net pressure head by 50m)
3. ε_{pipe} = (power out)/(power in) ..... 75% (Note: power out is the power available at the turbine, power in is the theoretically available power)
4. $P_T == \rho g Q h_n \varepsilon_T \varepsilon_G \varepsilon_{Tr} \varepsilon_{Coup}$ ..... 1.171 kW
5. $\varepsilon_{global} = P_T/P$ 59,7%
6. $P_T = 8 h_n Q$ ..... 1.200 kW

Pumps

Introduction

Once the capacity of pump needed is determined, the appropriate pump type should be chosen. There are many different types of pumps for water systems, and choosing the wrong one will lead to unsatisfactory water service.

There are basically two categories of pumps. Those used for lifting water from depths less 6.7 m and those used for lifting water from greater depths. Regardless of the pump type, however, each kind pulls water by suction from the well, and pushes the water by pressure into the distribution system. With this in mind, why then is there a distinction between deep and shallow wells?



In lifting water by suction all the pump is doing is removing air from the inside of the pipe. As this occurs, the pressure inside the pipe drops below its original pressure of atmospheric pressure. Since the pressure acting on the water outside the suction pipe is atmospheric, and the pressure above the water inside the suction pipe is less than atmospheric, the water in the well rises as a result of the difference in pressures.

As the pump continues to operate more and more air is removed from inside the suction pipe further lowering the pressure, until the atmospheric pressure finally forces the water in the suction pipe to the level of the pump. Assuming the pump in use is 100% efficient, the maximum height to which the atmospheric pressure could push the water is around 10m. Most real pumps, however, can only raise the water 4.6m to 8.5m (15 to 28 feet). To be able to lift water from depths greater than 8.5m (28 feet), the pump needs to be lowered until it is within 8.5m from the water level. Then it can lift the water to its level and push it to the surface. Shallow pumps are placed on the ground surface.

The following is a list of the different pump types.

- Shallow Reciprocating or Piston
- Deep Reciprocating or Piston
- Jet or Ejector
- Centrifugal
- Turbine Multistage
- Submersible Multistage
- Helical Rotor

Pumps can be broadly categorized into two classifications:

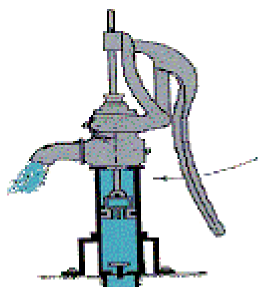
Categories of pumps

Positive displacement

Displacement pumps are typified by piston-and-cylinder pumps. A fixed amount of the pumped fluid is trapped and then ejected with a higher pressure and a velocity. Another fixed amount of fluid is then trapped, and so on. Examples of displacement pumps are rotary gear pumps, shallow piston pumps deep piston pumps, and diaphragm pumps.

Illustration of a positive displacement pump –Shallow Piston pump as an example

A piston pump uses the up and down or back and forth movement of a piston to displace water in a cylinder . As a piston is driven in one direction, water fills the chamber behind it. The water is forced into the system when the piston reverses directions. Flow of water into and out of the chamber is controlled by valves.



Manually Operated Surface-Type Piston Pump

Shallow Piston pumps can be used up to 6.7m (22 feet). Pump capacity depends on cylinder size and strokes per minute. The pressures it can produce are limited by the strength of the pumping equipment and motor horsepower.

Advantages of this type of pump are that it can pump small amounts of sand. It can also be installed over small diameter wells.

Disadvantages are that it causes a pulsating discharge, and may cause noise and vibrations.

Continuous flow

Continuous flow pumps are typified by the compressors used for jet engines. Flow is continuous and the fluid inertia is used to keep the pump going. Examples of continuous flow pumps are centrifugal flow and axial flow rotary pumps.

In this lecture we will concentrate on centrifugal pumps only but students are advised to read on their own on other types of pumps.

Centrifugal

Introduction

The operating principle of the centrifugal pump can be illustrated by considering the effect of swinging a bucket of water around in a circle of water at the end of a rope. The force pushing the water against the bottom of the bucket is centrifugal force. If a hole were cut in the bottom of the bucket, water would flow through the hole. Further, if an intake pipe where connected to an air tight cover over the top of the bucket, the flow of water out the hole would result in the

development of a partial vacuum inside the bucket.

This vacuum would bring water into the bucket from a source at the other end of the intake. In this way, continuous flow from the source and out through the bucket would be established.

In terms of real centrifugal pumps, bucket and lid correspond the pump casing, the hole and intake pipe correspond to the intake and discharge of the pump, and the rope and arm perform the functions of the impeller.

Advantages

Produces a smooth and even flow. Some types pump some sand. Centrifugal pumps are also usually reliable with a good service life

Disadvantages

Centrifugal pumps lose their prime easily, and their efficiency depends upon on operating under design heads and speed

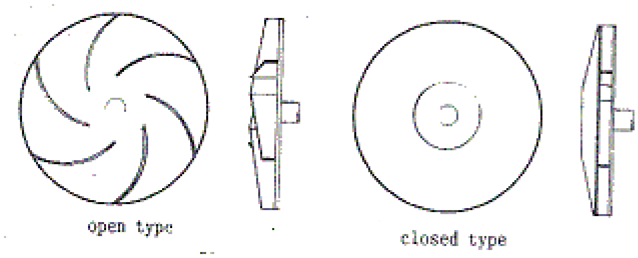
Remember:

Pumps that operate by suction cannot pump air. As a result the pump and pipe must be kept full of water, or primed.

Construction

The main parts of a centrifugal pump are the rotating part, the impeller, and the outer stationary part, the casing. The impeller is usually a casting, but can also be fabricated for large pumps. It consists of a disk with vanes on it.

Refer to diagram below

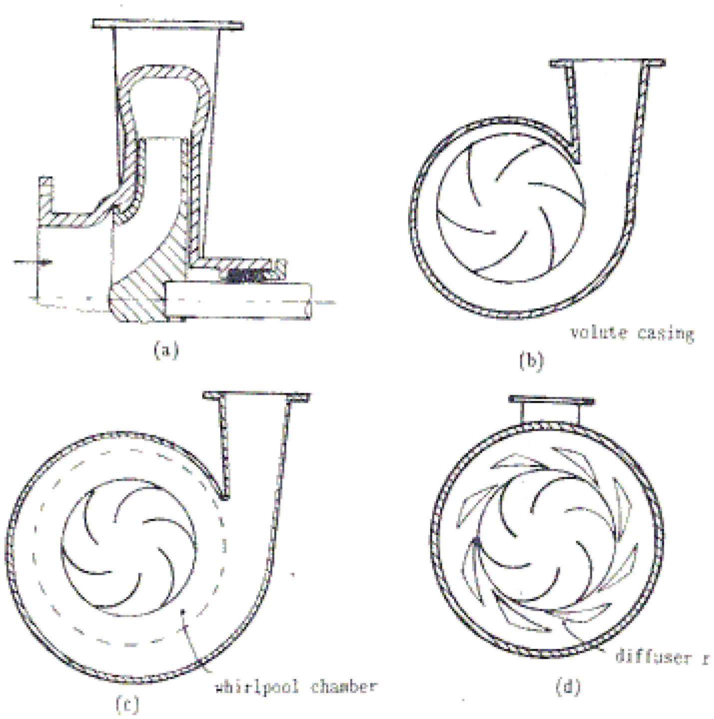


Impeller types

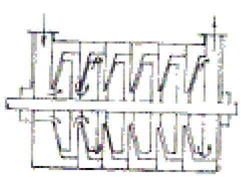
When the impeller rotates, kinetic energy is transferred by the vanes to the liquid while the liquid is flowing outwards. Most pumps have a single inlet on one side of the pump. Low pressure, high volume pumps sometimes have symmetrical flow with an inlet both sides of the pumps and a common exit .

The water is collected by the volute casing around the impeller which ends in a single discharge pipe. The shape of the casing must be such that it changes the high speed of the liquid into a lower speed at a higher pressure. The casing can be in the shape of a volute so that the constantly increasing cross sectional area decelerates the liquid. Refer figure (b). The casing can also form a concentric space between the impeller and the volute, called the whirlpool chamber in which the water rotates as a free vortex while also flowing outwards. The velocity of the water decreases according to free vortex theory on its way to the outer edge, and the pressure increases to the outside. The slower moving fluid is then collected in the volute casing. Refer figure (c).

The whirl pool chamber can also be fitted with vanes guiding the liquid as if it is flowing in a diverging channel. The guide vanes therefore limit the amount of energy sapping turbulence and acts as a diffuser. Refer figure (d).



Parts of centrifugal pumps

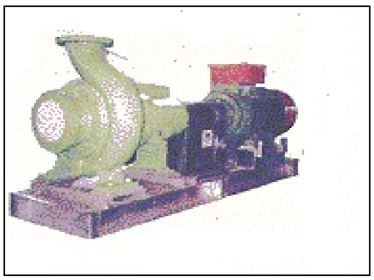


To obtain a compact high pressure pump, several impeller-volute assemblies can be combined with a common shaft to form a multi-stage pump. The liquid then passes from a diffuser outlet directly to the impeller inlet of the next stage.

Pressure heads of several hundreds of metres are possible with multi-stage pumps. Because these pumps also look like turbines (with reverse flow direction) they are sometimes referred to as turbine pumps.

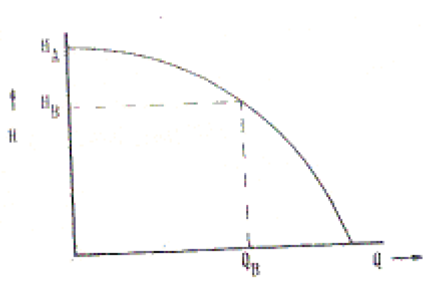
Performance of centrifugal pumps

Single



The pressure head that can be delivered by a pump is not constant but depends on the rotational speed as well as the flow rate through the pump. The theory of that is beyond the scope of this lecture -we suffice by investigating the typical characteristic curves of pumps and how it is applied in simple piping systems.

The relationship between the pressure head which a centrifugal pump can deliver, and the flow rate through the pump, is given by a pump curve or characteristic curve as shown below. Such an H-Q curve is based on experimental results .



The characteristic H-Q curve of a centrifugal pump.

If there is not any flow through the pump, because for instance a valve in the discharge pipe has been closed, then the pump will deliver a pressure head H_A. If the valve is opened so that the flow rate of Q_B results, the pump will be delivering at a pressure H_B etc. The pressure head H_B is the total pressure head which the pump must develop in order to overcome the external flow resistance. The external flow resistance consists of the following factors:

- A static pressure head H_{stat} which is the sum of the suction head and the delivery head.
- A friction head H_f which is the sum of the friction heads in the suction pipe and in the delivery pipe. In both cases the losses for the specific pipe can be calculated by the Darcy formula , or by other methods. The resistance of pipe fittings must also be included here.

    - The velocity head or dynamic pressure head H_v of the fluid in the discharge pipe, $H_v = V^2/2g$. This kinetic energy of the water in the discharge pipe has been supplied by the pump. Note, the kinetic energy in the suction pipe is not regarded as a loss because it can be regarded as if it is used to obtain the velocity head in the discharge pipe.

The sum of the above losses form the system resistance H_{sys}

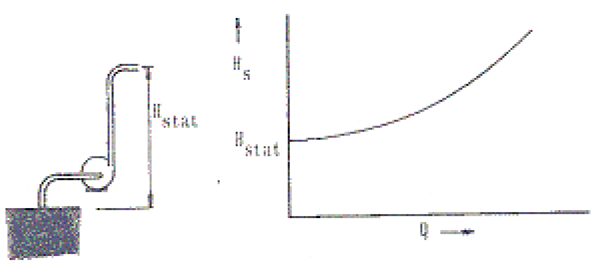
$H_{sys} = H_{stat} + H_f + H_v$

Because H_f as well as H_v depends on the square of the velocity and

thus also the square of the flow rate, we can change equation to

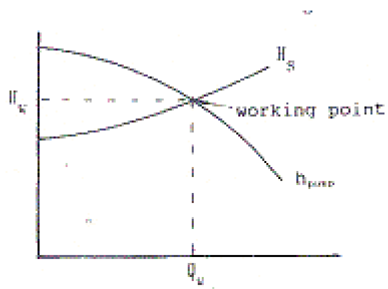
$H_{sys} = H_{stat} + kQ^2$

where k is a constant which depends on the pipe details. If we now plot the above equation we get the following curve.



The system resistance curve

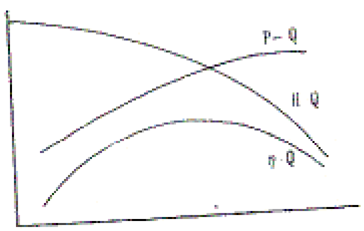
By plotting the system curve and the pump curve on the same axis, we get the intersection of the two which we called the working point of this pump piping system.



The power required to drive the pump as well as the efficiency curves are usually also plotted against Q from experimental results. The power and the efficiency are of course related by the following formula.

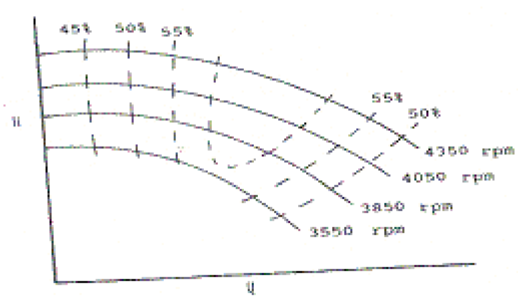
$$P = \frac{\rho g Q_w H_w}{\eta_w}$$

Where η_w is the efficiency at the working point - this is the efficiency of the pump at a flow rate of Q_w. Typical H- Q, P-Q and η - Q curves are shown below.



Characteristic curves of H, p and against Q.

A centrifugal pump can be used over a wide range of rotational speeds and all the information of various H- Q curves for various rotational speeds as well as efficiency values are often shown on one graph.



To obtain the working point from such a graph the system H- curve is simply plotted over such a graph. The intersection with the H-Q curve for the speed that is used gives the working point. The pump efficiency is obtained by using the efficiency curves and estimating an efficiency value as best as possible. The input power can then be calculated by the power equation.

Another factor that determines the pump characteristics is the impeller size. A whole range of impellers of various diameters can often be fitted to a pump. If the power required by a pump is to high for the electric motor driving the pump, the impeller diameter can be machined down to a smaller size, to reduce the input power.

Manufacturers therefore supply graphs giving the H-Q curve for various impeller diameters. The bigger the impeller the higher the pressure head and flow rate which the pump can deliver. The power required will also be more.

CLASS EXAMPLE

A test on a centrifugal pump which rotates at 1400 r/min, supplied the following results.

Discharge Q m^3/s)	0	0.079	0.158	0.238	0.317
Total pressure head H(m)	44.5	42.3	37.4	28.6	15.3

The pump is connected to a suction pipe, length 2,5 m and a discharge pipe, length 40 m, each with diameter 200 mm. Water is pumped from a level 1,5 m below the pump centre line and the discharge is at a level 16,5 m above the pump. Assume f = 0,006 and determine the flow rate in m^3/h, head in m and the pump power [774 m^3/h, 32.5m].

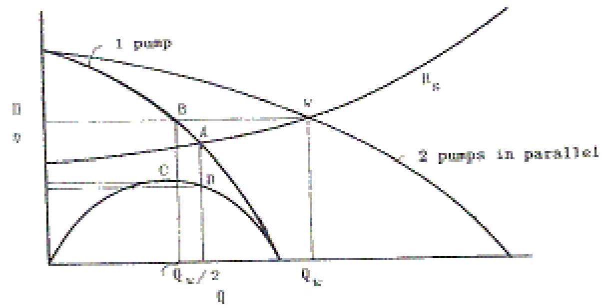
Parallel

Sometimes it could be advantageous to operate a few pumps in parallel rather than one single large pump. By having more than one pump, one of the pumps could be a standby pump giving a more dependable system. To prevent back flow through the pumps which are not working, one way valves are usually installed just after the pumps.

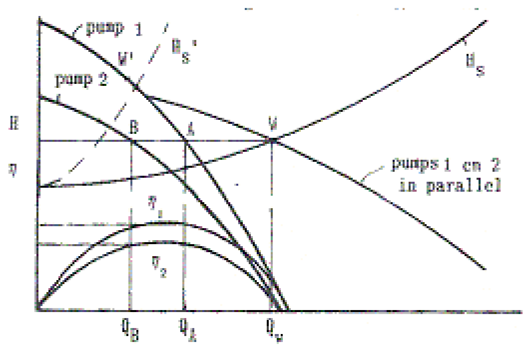
If two pumps have identical H-Q curves, the graphs will look as shown below when operated in parallel.

Characteristic curves of identical pumps in parallel

The discharge of one pump at a certain head is doubled and the intersection of the curve thus obtained with the system curve gives the working point. The flow through each pump is therefore $Q_w/2$. The efficiency of the pumps can be read from the -Q curve and is . If only one pump is operated its working point will be at A and the efficiency of the pump will be.



The same method is used for pumps which do not have the same characteristic curves. Each time the flow rates at a certain pressure head are added together.



Characteristic curves of non identical pumps in parallel

Pump 1 will deliver Q_A and pump 2 will deliver Q_B ($Q_A + Q_B$ is of

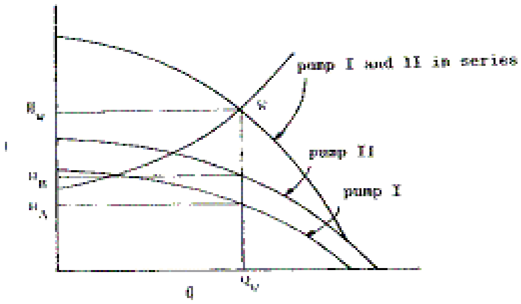
course equal to Q_w). The efficiency of each pump must be obtained from the efficiency -Q curve at the flow rate corresponding to the flow rate through that specific pump.

If a valve in a piping system is partially closed the system curve will rise sharply as indicated by the dotted line H'_s .Then the delivery of pump 2 will stop and only pump 1 will be delivering at its working point W'.

Series

If the system pressure head is to high for a single pump, a multi- stage pump can be used, which is in effect several pumps in series. Alternatively separate pumps can be connected in series or separate pumps can be installed as booster pumps at various locations along a long pipeline. The latter method has the advantage that the pipe pressure is distributed more evenly, the disadvantage being that in the case of long pipelines the controls and power supplies are not at a single central place but at various places along the line.

If two pumps are connected in series in a pipeline, we obtain the new characteristic curve by adding the pressure heads of the different pumps for a certain flow rate.



The flow rate through each pump is the same. Pump I provides a head H_A and pump II provides a total head H_B. H_A + H_B gives H_w, the total pressure increase by the two pumps. The efficiency of each pump is obtained from its own -Q curve at the flow rate Q_w

($Q_w = Q_A = Q_B$) .

Example

The characteristic curves of a certain pump can be plotted from the following test results.

Q (m3/s)	0	0.01	0.02	0.03	0.04
H (m)	22.5	21	18	12.5	3
(%)	0	66	81	70	20

The pump is connected to a water piping system head sys = 16.5 + 11550Q2 with H in m and Q in m3/s. Determine the flow rate and power required if:

(a) a single pump is connected in the system,

(b) two identical pumps are connected in parallel in the system,

(c) two identical pumps are connected in series in the system.

Suction considerations

Since most pumping problems are experienced at suction side, and are often insufficiently understood, an explanation of the various factors will be given in this lecture.

Referring to the figure A below the situation is that the suction is from a source either greater or less than atmospheric. Moreover the suction is flooded, i.e., the liquid level is above the pump centre line.

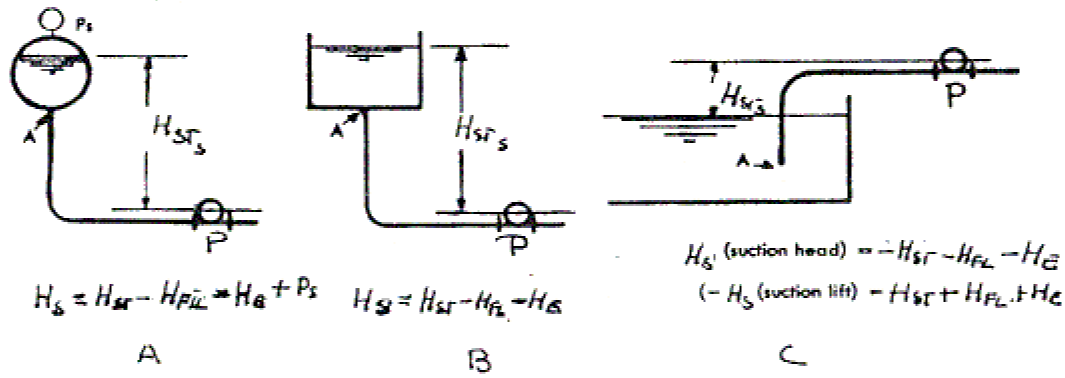


Fig B shows the situation when source is under atmospheric pressure and with flooded suction.

Fig C is more or less the same as fig B. However the source is also below pump level.

H_s = Suction head

Ps = Gauge reading

H_{sts} = Static head at suction

H_{fl} = Friction losses

He = Entrance losses

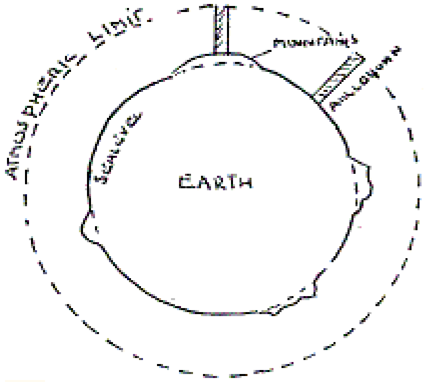
NET POSITIVE SUCTION HEAD

In order to make a pump work, it needs a certain amount of energy at its suction. This is expressed in m liquid column and is called the net positive suction head required by the pump (NPSH). This energy is depending on the type of pump and the remaining pressure at the suction must of course be greater than the vapour pressure of this liquid at its pumping temperature.

Should the pressure become lower than the vapour pressure at the liquids temperature, this would flash into vapour and the pump would stop to operate, or in more marginal conditions cavitate.

When looking at figure C, what would make the liquid in the reservoir to flow to the pump?

The only energy available at the liquid surface is the barometer pressure. We have seen in that the atmospheric pressure at sea level can lift a column of water 10,3m. This is the pressure exerted by the mass of the air on it.



From the figure above it is clear that at heights above sea level, there is a shorter column, hence the atmospheric pressure is lower

Turning now again to fig C, we can see that the liquid has to be lifted a height of H_{st} to the pump suction. Thus, disregarding the losses the head at the pump- suction would be:

B $-H_{sts}$ in which B = is the atmospheric or Barometric pressures. As we know a liquid column (head) represents a certain amount of energy.

But we also have to take into consideration the friction losses in the suction system, so we have the following energy available at the pump suction $B-H_{sts}$ $-H_{fl}$ (entry losses included in H_{fl}). The remaining pressure (energy) is of course a vacuum, and must be greater than the vapour pressure of the liquid at the pumping temperature. We call the resulting energy (or head) the NET POSITIVE SUCTION HEAD AVAILABLE ($NPSH_A$) , i.e

$NPSH_A = B - H_{sts} - H_{fl} - v_p$

v_p is the vapour pressure.

Now, as mentioned before, the pump requires a certain energy at suction to

operate, i.e., $NPSH_R$ and it will be clear that $NPSH_A$ $NPSH_R$.

Now what will happen if we have a suction lift H_{sts} which is so high that the vacuum at suction is lower than the vapour pressure?

Part of the liquid will flash into vapour forming bubbles in the liquid.

CLASS EXAMPLE:

A boiler feed pump has an $NPSH_R$ at duty point of 3m. The barometric pressure is 9mW.G. and the water in the hot well is 90°C. The friction loss in the suction line can be estimated at 0,5m. What is the minimum suction head?

Pump calculations

http://www.tasonline.co.za/toolbox/index.htm

Centrifugal pump troubles and causes

Symptoms and possible causes

Pump does not deliver water:

- Pump not primed.
- Pump or suction pipe not completely filled with liquid.
- Suction lift too high.
- Insufficient margin between suction pressure and vapor pressure.
- Air pocket in suction line.
- Inlet of suction pipe insufficiently submerged.
- Speed too low.
- Wrong direction of rotation.
- Total head of system higher than design head of pump.
- Parallel operation of pumps unsuitable for such operation.
- Foreign matter in impeller.

Insufficient capacity delivered:

- Pump or suction pipe not completely filled with liquid.
- Suction lift too high.
- Insufficient margin between suction pressure and vapor pressure.
- Excessive amount of air or gas in liquid.
- Air pocket in suction line.
- Air leaks into suction line.
- Air leaks into pump through stuffing boxes.
- Foot valve too small.

- Foot valve partially clogged.
- Inlet of suction pipe insufficiently submerged.
- Speed too low.
- Total head of system higher than design head of pump.
- Viscosity of liquid differs from that for which designed.
- Parallel operation of pumps unsuitable for such operation.
- Foreign matter in impeller.
- Wearing rings worn.
- Impeller damaged.
- Casing gasket defective permitting internal leakage.

Insufficient pressure developed:

- Excessive amount of air or gas in liquid.
- Speed too low.
- Wrong direction of rotation.
- Total head of system higher than design head of pump.
- Viscosity of liquid differs from that for which designed.
- Parallel operation of pumps unsuitable for such operation.
- Wearing rings worn.
- Impeller damaged.
- Casing gasket defective permitting internal leakage.

Pump loses prime after starting:

- Pump or suction pipe not completely filled with liquid.
- Suction lift too high.
- Excessive amount of air or gas in liquid.
- Air pocket in suction line.
- Air leaks into suction line.
- Air leaks into pump through stuffing boxes.
- Inlet of suction pipe insufficiently submerged.
- Water-seal pipe plugged.

- Seal cage improperly located in stuffing box, preventing sealing fluid entering space to form the seal.

Pump requires excessive power:

- Speed too high.
- Wrong direction of rotation.
- Total head of system higher than design head of pump.
- Total head of system lower than pump design head.
- Specific gravity of liquid different from design.
- Viscosity of liquid differs from that for which designed.
- Foreign matter in impeller.
- Misalignment.
- Shaft bent.
- Rotating part rubbing on stationary part.
- Wearing rings worn.
- Packing improperly installed.
- Incorrect type of packing for operating conditions.
- Gland too tight resulting in no flow of liquid to lubricate packing.

Stuffing box leaking excessively:

- Seal cage improperly located in stuffing box, preventing sealing fluid entering space to form the seal.
- Misalignment.
- Shaft bent.
- Shaft or shaft sleeves worn or scored at the packing.
- Packing improperly installed.
- Incorrect type of packing for operating conditions.
- Shaft running off center because of worn bearings or misalignment.
- Rotor out of balance resulting in vibration.
- Failure to provide cooling liquid to water-cooled stuffing boxes.
- Excessive clearance at bottom of stuffing box between shaft and casing, causing packing to be forced into pump interior.
- Dirt or grit in sealing liquid, leading to scoring of shaft or shaft sleeve.

Packing has short life:

- Water-seal pipe plugged.
- Seal cage improperly located in stuffing box, preventing sealing fluid entering space to form the seal.
- Misalignment.
- Shaft bent.
- Bearings worn.
- Shaft or shaft sleeves worn or scored at the packing.
- Packing improperly installed.
- Incorrect type of packing for operating conditions.
- Shaft running off center because of worn bearings or misalignment.
- Rotor out of balance resulting in vibration.
- Gland too tight resulting in no flow of liquid to lubricate packing.
- Failure to provide cooling liquid to water-cooled stuffing boxes.
- Excessive clearance at bottom of stuffing box between shaft and casing, causing packing to be forced into pump interior.
- Dirt or grit in sealing liquid, leading to scoring of shaft or shaft sleeve.

Pump vibrates or is noisy:

- Pump or suction pipe not completely filled with liquid.
- Suction lift too high.
- Insufficient margin between suction pressure and vapor pressure.
- Foot valve too small.
- Foot valve partially clogged.
- Inlet of suction pipe insufficiently submerged.
- Operation at very low capacity.
- Foreign matter in impeller.
- Misalignment.
- Foundations not rigid.
- Shaft bent.

- Rotating part rubbing on stationary part.
- Bearings worn.
- Impeller damaged.
- Shaft running off center because of worn bearings or misalignment.
- Rotor out of balance resulting in vibration.
- Excessive thrust caused by a mechanical failure inside the pump or by the failure of the hydraulic balancing device, if any.
- Excessive grease or oil in antifriction-bearing housing or lack of cooling, causing excessive bearing temperature.
- Lack of lubrication.
- Improper installation of antifriction bearings (damaged during assembly, incorrect assembly of stacked bearings, use of unmatched bearings as a pair, etc)
- Dirt getting into bearings.
- Rusting of bearings due to water getting into housing.
- Excessive cooling of water –cooled bearing resulting in condensation in the bearing housing of moisture from the atmosphere.

Bearings have short life:

- Misalignment.
- Shaft bent.
- Rotating part rubbing on stationary part.
- Bearings worn.
- Shaft running off center because of worn bearings or misalignment.
- Rotor out of balance resulting in vibration.
- Excessive thrust caused by a mechanical failure inside the pump or by the failure of the hydraulic balancing device, if any.
- Excessive grease or oil in antifriction-bearing housing or lack of cooling, causing excessive bearing temperature.
- Lack of lubrication.
- Improper installation of antifriction bearings (damaged during assembly, incorrect assembly of stacked bearings, use of unmatched bearings as a pair, etc)
- Dirt getting into bearings.
- Rusting of bearings due to water getting into housing.

- Excessive cooling of water –cooled bearing resulting in condensation in the bearing housing of moisture from the atmosphere.

Pump overheats and seizes:

- Pump not primed.
- Insufficient margin between suction pressure and vapor pressure.
- Operation at very low capacity.
- Parallel operation of pumps unsuitable for such operation.
- Misalignment.
- Rotating part rubbing on stationary part.
- Bearings worn.
- Shaft running off center because of worn bearings or misalignment.
- Rotor out of balance resulting in vibration.
- Excessive thrust caused by a mechanical failure inside the pump or by the failure of the hydraulic balancing device, if any.

Further Reading

http://hyperphysics.phy-astr.gsu.edu/hbase/surten.html#c2

http://www.flowgate-projects.com/

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